1 Find the term independent of \( x \) in the expansion of \( \left(x - \frac{1}{x^2}\right)^9 \). \([3]\)

2 Points \( A, B \) and \( C \) have coordinates \((2, 5), (5, -1)\) and \((8, 6)\) respectively.

(i) Find the coordinates of the mid-point of \( AB \). \([1]\)

(ii) Find the equation of the line through \( C \) perpendicular to \( AB \). Give your answer in the form \( ax + by + c = 0 \). \([3]\)

3 Solve the equation \( 15 \sin^2 x = 13 + \cos x \) for \( 0^\circ \leq x \leq 180^\circ \). \([4]\)

4 (i) Sketch the curve \( y = 2 \sin x \) for \( 0 \leq x \leq 2\pi \). \([1]\)

(ii) By adding a suitable straight line to your sketch, determine the number of real roots of the equation

\[ 2\pi \sin x = \pi - x. \]

State the equation of the straight line. \([3]\)

5 A curve has equation \( y = \frac{1}{x - 3} + x \).

(i) Find \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \). \([2]\)

(ii) Find the coordinates of the maximum point \( A \) and the minimum point \( B \) on the curve. \([5]\)

6 A curve has equation \( y = f(x) \). It is given that \( f'(x) = 3x^2 + 2x - 5 \).

(i) Find the set of values of \( x \) for which \( f \) is an increasing function. \([3]\)

(ii) Given that the curve passes through \((1, 3)\), find \( f(x) \). \([4]\)
The diagram shows the function \( f \) defined for \( 0 \leq x \leq 6 \) by
\[
\begin{align*}
x & \mapsto \frac{1}{2}x^2 \quad \text{for} \quad 0 \leq x \leq 2, \\
x & \mapsto \frac{1}{2}x + 1 \quad \text{for} \quad 2 < x \leq 6.
\end{align*}
\]

(i) State the range of \( f \). [1]

(ii) Copy the diagram and on your copy sketch the graph of \( y = f^{-1}(x) \). [2]

(iii) Obtain expressions to define \( f^{-1}(x) \), giving the set of values of \( x \) for which each expression is valid. [4]

The diagram shows a rhombus \( ABCD \). Points \( P \) and \( Q \) lie on the diagonal \( AC \) such that \( BPD \) is an arc of a circle with centre \( C \) and \( BQD \) is an arc of a circle with centre \( A \). Each side of the rhombus has length 5 cm and angle \( BAD = 1.2 \) radians.

(i) Find the area of the shaded region \( BPDQ \). [4]

(ii) Find the length of \( PQ \). [4]
9  (a) A geometric progression has first term 100 and sum to infinity 2000. Find the second term. [3]

(b) An arithmetic progression has third term 90 and fifth term 80.

   (i) Find the first term and the common difference. [2]

   (ii) Find the value of \( m \) given that the sum of the first \( m \) terms is equal to the sum of the first \( (m + 1) \) terms. [2]

   (iii) Find the value of \( n \) given that the sum of the first \( n \) terms is zero. [2]

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The diagram shows triangle \( OAB \), in which the position vectors of \( A \) and \( B \) with respect to \( O \) are given by

\[
\overrightarrow{OA} = 2i + j - 3k \quad \text{and} \quad \overrightarrow{OB} = -3i + 2j - 4k.
\]

\( C \) is a point on \( OA \) such that \( \overrightarrow{OC} = p \overrightarrow{OA} \), where \( p \) is a constant.

   (i) Find angle \( AOB \). [4]

   (ii) Find \( \overrightarrow{BC} \) in terms of \( p \) and vectors \( i, j \) and \( k \). [1]

   (iii) Find the value of \( p \) given that \( BC \) is perpendicular to \( OA \). [4]
The diagram shows parts of the curves $y = 9 - x^3$ and $y = \frac{8}{x^3}$ and their points of intersection $P$ and $Q$. The $x$-coordinates of $P$ and $Q$ are $a$ and $b$ respectively.

(i) Show that $x = a$ and $x = b$ are roots of the equation $x^6 - 9x^3 + 8 = 0$. Solve this equation and hence state the value of $a$ and the value of $b$. [4]

(ii) Find the area of the shaded region between the two curves. [5]

(iii) The tangents to the two curves at $x = c$ (where $a < c < b$) are parallel to each other. Find the value of $c$. [4]