This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners’ meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2009 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.
Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0. B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF  Any Equivalent Form (of answer is equally acceptable)
AG   Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD  Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO  Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO  Correct Working Only often written by a ‘fortuitous’ answer
ISW  Ignore Subsequent Working
MR   Misread
PA   Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS  See Other Solution (the candidate makes a better attempt at the same question)
SR   Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.
1 \[3 \tan(2x + 15^\circ) = 4\] 
\[\tan(2x + 15^\circ) = 1\frac{1}{3}\] 
Sets the bracket to \(\tan^{-1}(1\frac{1}{3})\) 
\[2x + 15 = 53.13^\circ\] or \[233.13^\circ\] 
\[\rightarrow x = 19.1^\circ\] or \[109.1^\circ\] 
M1 M1 A1 A1 \[\sqrt{\frac{3}{2}}\] 
Removes the “3” first by division. 
Looks up \(\tan^{-1}1\frac{1}{3}\), then uses bracket 
co. \(\sqrt{\frac{3}{2}}\) for \((90 + 1^\text{st}\) answer) and no other answers in the range.

2 
B1 
1 complete oscillation \(0 \rightarrow \pi\) 
B1 
Range from \(-3\) to \(3\) 
B1 
All correct (V shape B0) 
B1 
Line correct.

3 (i) \[2 - x \right)^6\] 
\[64 - 192x + 240x^2\] 
3 \(\times\) B1 
[3] 
co 
Allow \(2^6\). 

(ii) \[(1 + 2x + \alpha x^2)(2 - x)^6\] 
Coef of \(x^2 = 240 - 384 + 64\alpha\) 
Equates to 48 
\[\rightarrow \alpha = 3\] 
M1 M1 A1 
[3] 
Considers at least 2 terms in \(x^2\). 
Considers exactly 3 terms + solution 
co

4 \[y = x^4 + 4x + 9\] 
(i) Differential = \(4x^3 + 4\) 
Sets to 0 + solution \(\rightarrow (-1, 6)\) 
2\(^{\text{nd}}\) differential = \(12x^2\) 
Positive, \(\rightarrow\) Minimum 
B1 M1 A1 
[4] 
co 
Differentiates and sets to 0. co. 
Statement only.

(ii) \[A = \left[\frac{x^5}{5} + 2x^2 + 9x\right]\] 
Limits from 0 to 1 \(\rightarrow 11.2\) 
B1 M1 A1 
[3] 
co. 
Value at “1” – value at “0” in integral of \(y\).

5 \[r = 6 \, \text{cm}\] 
(i) \[AB = \sqrt{(6^2 + 6^2)} = \sqrt{72}\] 
Angle \(BAD = \frac{\pi}{4}\) or \(45^\circ\) 
Arc length = \(\sqrt{72} \times \frac{\pi}{4} = 6.66(7)\) 
B1 M1 A1 
[4] 
Use of Pythagoras – or trig (8.5 ok) 
In degrees or radians 
Use of \(s=r\theta\) with \(\theta\) in radians only – or correct with degrees. Use of \(r = 6 \, \text{M0}\).

(ii) Sector area = \(\frac{1}{2} r^2 \theta = \frac{1}{2} \times 72 \times \frac{\pi}{4}\) 
Area of triangle = \(\frac{1}{2} \times 6 \times 6\) 
Shaded area = \(10.3\) or \(9\pi - 18\) 
M1 B1 A1 
[3] 
Use of \(\frac{1}{2} r^2 \theta\) with \(\theta\) in rad, and \(r \neq 6\). 
co
6 \( \frac{dy}{dx} = k - 2x \)

(i) At \( x = 2, m = (k - 4) \) \( x = 3 \)
\[
m = (k - 6)
\]
\[
(k - 4)(k - 6) = -1
\]
\[
\rightarrow k = 5
\]

Obtains either gradient as \( f(k) \).
Uses \( m_1m_2 = -1 \) with gradients \( f(k) \)
Soln of quadratic = 0. co
(watch for fortuitous answers)

(ii) \( y = kx - x^2 + c \)
Substitutes (4, 9)
\[
\rightarrow c = 5
\]

For integration without \( c \)
Realises need to substitute for \( x \) and \( y \)
co
(nb If \( k = 5 \) is fortuitous, loses last A1)

7 \( y = \frac{12}{x^2 + 3} \)

(i) \( \frac{dy}{dx} = -12(\alpha^2 + 3)^2 \times 2x \)

Without the “\( \times 2x \)”. For “\( \times 2x \)”.
Accept unsimplified answer

(ii) At \( x = 1, m = -\frac{3}{2} \)
\[ m \text{ of normal} = \frac{\sqrt{3}}{2} \]
Eqn of normal
\[ y - 3 = \frac{\sqrt{3}}{2} (x - 1) \]

Uses \( m_1m_2 = -1 \) ....algebraic ok.
Correct form of equation.
co unsimplified

(iii) \( \frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = -\frac{3}{2} \times 0.012 \)
\[ \rightarrow -0.018 \]

Correct link between differentials
correct formula used.

8 (i) \( 8 + 4d = 8r \)
\[ 8 + 7d = 8r^2 \]
Eliminates one of the variables
\[ \rightarrow 4r^2 - 7r + 3 = 0 \]
Solution
\[ \rightarrow r = \frac{7}{4} \]
\[ d = -\frac{1}{2} \]

co – but allow if \( a \) in place of 8.
co – but allow if \( a \) in place of 8.
Complete elimination of either \( r \) or \( d \).
Correct method of solution.
(assumes \( r = \frac{7}{4} \), give B1B1 for equations, B1 for \( d \))

(ii) \( S_\infty = \frac{a}{1-r} \rightarrow 32 \)

Correct formula used.

(iii) \( S_8 = 4(16 + 7d) \)
\[ = 50 \]
Correct formula used. \( 64 + 28d \) ok
co
9  \( \vec{OA} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}, \vec{OB} = \begin{pmatrix} 0 \\ 8 \\ -6 \end{pmatrix}, \vec{OC} = \begin{pmatrix} -2 \\ 5 \\ -2 \end{pmatrix} \)  

(i) Scalar product: \(-18 - 48\)  
\(-66 = |a| \cdot |b| \cos \theta\)  
|a| = 7 and |b| = 10  
\(\rightarrow\) Angle \(AOB = 160.5^\circ\)  
M1 Use of \(x_1x_2 + y_1y_2 + z_1z_2\)  
M1 Linking everything correctly  
M1 Correct modulus of either \(a\) or \(b\).  
A1 Co  
[4]

(ii) \(\vec{AC} = \vec{c} - \vec{a} = \begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}\)  
Modulus = 6  
Vector = \(5 \times \begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}\) or \(\begin{pmatrix} -20 \\ 10 \\ 20 \end{pmatrix}\)  
B1 Co, allow ±.  
M1 For modulus and multiplying by “5”  
A1 Co  
[3]

(iii) \(\begin{pmatrix} 2 \\ 3 - 6p \\ -6 + 8p \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 5 \\ -2 \end{pmatrix} = 0\)  
\(\rightarrow p = \frac{1}{2}\)  
B1 For \(\vec{OA} + p\vec{OB}\) as single vector.  
M1 Scalar product = 0.  
A1 Co (beware fortuitous answers)  
[3]
\[ f: x \mapsto 2x + 1, \quad x \in \mathbb{R}, \ x > 0 \]
\[ g: x \mapsto \frac{2x - 1}{x + 3}, \quad x \in \mathbb{R}, \ x \neq -3. \]

(i) \[ gf(x) = \frac{2(2x + 1) - 1}{2x + 1 + 3} = \frac{4x + 1}{2x + 4} \]
Equates to \( x \rightarrow 2x^2 = 1 \)
\[ x = \frac{1}{2} \sqrt{2} \]
M1 Must be \( g \), needs \( x \) replacing twice.
M1 Forms quadratic + solution
A1 Co. condone ±.

(ii) \[ f^{-1}(x) = \frac{1}{2}(x - 1) \]
To find \( g^{-1}(x) \), make \( x \) the subject
Order must be correct
\[ g^{-1}(x) = \frac{1 - 3x}{x - 2} \text{ or } \frac{1 + 3x}{2 - x} \]
A1 Co – must be \( f(x) \).

(iii) \[ x = x^2 + x + 1 = 0 \]
\[ \frac{1 + 3x}{2 - x} = x \]
Looks at \( b^2 - 4ac \)
\[ \rightarrow \text{negative} \rightarrow \text{no roots.} \]
M1 Forms quadratic equation.
M1 Looks at discriminant or attempts to solve and finds \( \sqrt{\text{negative}} \). Co
A1

(iv) \[ \text{Correct } y = 2x + 1 \text{ on graph from (0, 1)} \]
\[ \text{Correct } y = \frac{1}{2}(x - 1) \text{ on graph from (1, 0)} \]
\[ \text{if –ve } x \text{ plotted, B1 s.c. for both} \]
B1 Shows or states or implies that \( f, f^{-1} \) are reflections in \( y = x \).

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