MARK SCHEME for the November 2003 question papers

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These mark schemes are published as an aid to teachers and students, to indicate the requirements of the examination. They show the basis on which Examiners were initially instructed to award marks. They do not indicate the details of the discussions that took place at an Examiners’ meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published Report on the Examination.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates’ scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the Report on the Examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2003 question papers for most IGCSE and GCE Advanced Level syllabuses.
Mark Scheme Notes

- Marks are of the following three types:

  M  Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A  Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B  Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0. B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking \( g \) equal to 9.8 or 9.81 instead of 10.
• The following abbreviations may be used in a mark scheme or used on the scripts:

AEF  Any Equivalent Form (of answer is equally acceptable)
AG   Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD  Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO  Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO  Correct Working Only – often written by a ‘fortuitous' answer
ISW  Ignore Subsequent Working
MR   Misread
PA   Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS  See Other Solution (the candidate makes a better attempt at the same question)
SR   Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

• MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √"marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

• PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.
November 2003

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 75

SYLLABUS/COMPONENT: 9709/01

MATHEMATICS
Pure Mathematics : Paper One
### 1

- **x(11-2x) = 12**
  - **2x²-11x+12=0**
  - Solution of quadratic
  - → \((1\frac{1}{2},8)\) and \((4,3)\)

  **Mark Scheme**
  - M1: Complete elimination of x, or of y.
  - A1: Correct quadratic. (or \(y^2-11y+24=0\))
  - DM1: Correct method of solution→2values
  - A1: All correct
  - (guesswork or TI B1 for one pair of values, full marks for both)

  **Score [4]**

### 2

- (i) **4s^4+5=7(1-s^2)** → **4x^2+7x-2=0**
- (ii) **4s^4+7s^2-2=0**
  - → \(s^2 = \frac{1}{4}\) or \(s^2 = -2\)
  - → \(\sin\theta = \pm\frac{1}{2}\)
  - → \(\theta = 30^\circ\) and \(150^\circ\)
  - and \(\theta = 210^\circ\) and \(330^\circ\)

  **Mark Scheme**
  - B1: Use of \(s^2+c^2=1\). Answer given.
  - M1: Recognition of quadratic in \(s^2\)
  - A1: For \(180^\circ - \text{“his value”}\)
  - A1√: For other 2 answers from “his value”, providing no extra answers in the range or answers from \(s^2=-1\)

  **Score [4]**

### 3

- (a) **a=60, n=48, S_n=3726**
  - S_n formula used
  - → \(d = 0.75\)
  - 3rd term = \(a+2d = 61.50\)

  **Mark Scheme**
  - M1: Correct formula (M0 if nth term used)
  - A1: Use of \(a+2d\) with his \(d\).
  - A1√: \(61.5\ ok.\)

  **Score [3]**

- (b) **a=6 ar =4**: \(r=\frac{3}{4}\)
  - **S_n = a/(1-r) = 18**

  **Mark Scheme**
  - M1: a, ar correct, and \(r\) evaluated
  - A1: Correct formula used, but needs \(r < 1\) for M mark

  **Score [3]**

### 4

- (i) **\(y = x^3 - 2x^2 + x\) (+c )**
  - (1,5) used to give \(c = 5\)

  **Mark Scheme**
  - B2,1,0: Co - unsimplified ok.
  - B1√: Must have integrated + use of \(x=1\)and \(y=5\)
  - for \(c\)

  **Score [3]**

- (ii) **3x^2-4x+1>0**
  - → end values of \(1\) and \(\frac{1}{3}\)
  - → \(x<\frac{1}{3}\) and \(x>1\)

  **Mark Scheme**
  - M1: Set to 0 and attempt to solve.
  - A1: Co for end values – even if \(<,>,=,\text{etc}\)
  - A1: Co (allow \(\leq\) and \(\geq\)). Allow \(1<x<\frac{1}{3}\)

  **Score [3]**

### 5

#### (i)
- **m of BC = \(\frac{1}{2}\)**
  - **Eqn BC y-6=\(\frac{1}{2}\)(x-4)**
  - **m of CD = -2**
  - **eqn CD y-5=-2(x-12)**

  **Mark Scheme**
  - B1: Correct form of eqn. √ on \(m=“\frac{1}{2}.”\)
  - M1: Use of \(m_1m_2=-1\)
  - A1: √ on his “\(\frac{1}{2}\)” but needs both M marks.

  **Score [5]**

#### (ii)
- **Sim eqns 2y=x+8 and y+2x=29**
  - → \(C (10,9)\)

  **Mark Scheme**
  - M1: Method for solving
  - A1: Diagram only for (ii), allow B1 for (10,9)

  **Score [2]**
6

(i) \(20 = 2r + r\theta\)
\[\theta = \frac{(20 / r) - 2}{2}\]

(ii) \(A = \frac{1}{2}r^2\theta\)
\[A = 10r - r^2\]

(iii)
\[\text{Cos rule } PQ^2 = 8^2 + 8^2 - 2 \times 8 \times 8 \cos 0.5\]
Or trig \(PQ = 2 \times 8 \sin 0.25\)
\[\rightarrow PQ = 3.96 \text{ (allow 3.95).}\]

7

(i) Height = 4

(ii) \(\text{MC} = 3i-6j-4k\)
\(\text{MN} = 6j - 4k\)

(iii) \(\text{MC} \cdot \text{MN} = -36 + 16 = -20\)
\[\text{MC} \cdot \text{MN} = \sqrt{61} \sqrt{52} \cos \theta\]
\[\rightarrow \theta = 111^\circ\]

8

(i) \(y = 72 \div (2x^2) \text{ or } 36 \div x^2\)
\[A = 4x^2 + 6xy\]
\[\rightarrow A = 4x^2 + 216 \div x\]

(ii) \(\frac{dA}{dx} = 8x - 216 \div x^2\)
\[= 0 \text{ when } 8x^3 = 216\]
\[\rightarrow x = 3\]

(iii) Stationary value = 108 cm²
\[\frac{d^2A}{dx^2} = 8 + 432 \div x^3\]
\[\rightarrow \text{Positive when } x = 3 \text{ Minimum.}\]
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<td></td>
<td>A AND AS LEVEL – NOVEMBER 2003</td>
<td>9709 1</td>
</tr>
<tr>
<td>(i) dy/dx = -24/(3x+2)^2</td>
<td>M1A1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eqn of tangent y-1=-⅜(x-2)</td>
<td>M1A1√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cuts y=0 when x=4⅔</td>
<td>M1A1</td>
<td></td>
<td>[6]</td>
</tr>
<tr>
<td>Area of Q = ½×2⅔×1 = 3/4</td>
<td>M1A1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use of line form with dy/dx. Must use calculus. √ on his dy/dx. Normal M0.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Needs y=0 and ½bh for M mark. (be sure fortuitous answers)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) Vol =π ∫ y²dx =π [64(3x+2)]²dx</td>
<td>M1</td>
<td></td>
<td>[5]</td>
</tr>
<tr>
<td>=π [- 64(3x+2)⁻¹ ÷ 3]</td>
<td>A1A1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Limits from 0 to 2</td>
<td>DM1</td>
<td></td>
<td></td>
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<tr>
<td>→ 8π</td>
<td>A1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uses ∫ y² + some integration → (3x+2)⁴.</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>A1 without the ÷3. A1 for ÷3 and π</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correct use of 0 and 2. DMO if 0 ignored.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Co. Beware fortuitous answers.</td>
<td></td>
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<tr>
<td>10</td>
<td></td>
<td>A AND AS LEVEL – NOVEMBER 2003</td>
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<tr>
<td>(i) fg(x) = g first, then f</td>
<td>M1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>= 8/(2-x) – 5 = 7</td>
<td>DM1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>→ x = 1 1/3</td>
<td>A1</td>
<td></td>
<td>[3]</td>
</tr>
<tr>
<td>( or f(A)=7, A = 6, g(x) = 6, → x = 1 1/3 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) f⁻¹ = ½(x+5)</td>
<td>B1</td>
<td></td>
<td>[3]</td>
</tr>
<tr>
<td>Makes y the subject y = 4÷(2-x)</td>
<td>M1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>→ g⁻¹ = 2 – ( 4÷x)</td>
<td>M1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(iii) 2-4/x =½ (x+5)</td>
<td>M1</td>
<td></td>
<td>[3]</td>
</tr>
<tr>
<td>→ x²+x+8=0</td>
<td>M1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use of b²-4ac → Negative value</td>
<td>A1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>→ No roots.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(iv)</td>
<td>B1</td>
<td></td>
<td>[3]</td>
</tr>
<tr>
<td>Sketch of f</td>
<td>B1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sketch of f⁻¹</td>
<td>B1</td>
<td></td>
<td></td>
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<tr>
<td>Evidence of symmetry about y=x.</td>
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November 2003

GCE AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/02

MATHEMATICS
Pure Mathematics : Paper Two
1 **EITHER:** State or imply non-modular inequality e.g. \(-2 < 8-3x < 2\), or corresponding equation or pair of equations

M1

Obtain critical values 2 and 3 \(\frac{1}{3}\)

A1

State correct answer \(2 < x < 3\frac{1}{3}\)

A1

**OR:**
State one critical value (probably \(x = 2\)), from a graphical method or by inspection or by solving a linear equality or equation

B1

State the other critical value correctly

B1

State correct answer \(2 < x < 3\frac{1}{3}\)

B1

[3]

2 State or imply at any stage \(\ln y = \ln k - x \ln a\)

B1

Equate estimate of \(\ln y\) - intercept to \(\ln k\)

M1

Obtain value for \(k\) in the range 9.97 ± 0.51

A1

Calculate gradient of the line of data points

M1

Obtain value for \(a\) in the range 2.12 ± 0.11

A1

[5]

3 (i) **EITHER:** Substitute \(-1\) for \(x\) and equate to zero

M1

Obtain answer \(a=6\)

A1

**OR:** Carry out complete division and equate remainder to zero

M1

Obtain answer \(a=6\)

A1

[2]

(ii) Substitute 6 for \(a\) and either show \(f(x) = 0\) or divide by \((x - 2)\) obtaining a remainder of zero

B1

**EITHER:** State or imply \((x + 1)(x - 2) = x^2 - x - 2\)

B1

Attempt to find another quadratic factor by division or inspection

M1

State factor \((x^2 + x - 3)\)

A1

**OR:** Obtain \(x^3 + 2x^2 - 2x - 3\) after division by \(x + 1\), or \(x^3 - x^2 - 5x + 6\) after division by \(x - 2\)

B1

Attempt to find a quadratic factor by further division by relevant divisor or by inspection

M1

State factor \((x^2 + x - 3)\)

A1

[4]

4 (i) State answer \(R = 2\)

B1

Use trig formula to find \(\alpha\)

M1

Obtain answer \(\alpha = \frac{1}{3}\pi\)

A1

[3]
(ii) Carry out, or indicate need for, evaluation of \( \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) \)

M1*

Obtain, or verify, the solution \( \theta = \frac{7\pi}{12} \)

A1

Attempt correct method for the other solution in range

i.e. \( -\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) + \alpha \)

.M1(dep*)

Obtain solution \( \theta = \frac{1}{12} \pi : [M1A0 for \frac{25\pi}{12}] \)

A1

[4]

5 (i) Make recognisable sketch of \( y = 2^x \) or \( y = x^2 \), for \( x < 0 \)

B1

Sketch the other graph correctly

B1

[2]

(ii) Consider sign of \( 2^x - x^2 \) at \( x = -1 \) and \( x = -0.5 \), or equivalent

M1

Complete the argument correctly with appropriate calculations

A1

[2]

(iii) Use the iterative form correctly

M1

Obtain final answer \(-0.77\)

A1

Show sufficient iterations to justify its accuracy to 2 s.f., or show there is a sign change in the interval \((-0.775, -0.765)\)

A1

[3]

6 (i) State \( A \) is \((4, 0)\)

B1

State \( B \) is \((0, 4)\)

B1

[2]

(ii) Use the product rule to obtain the first derivative

M1(dep)

Obtain derivative \((4-x)e^x - e^x\), or equivalent

A1

Equate derivative to zero and solve for \( x \)

M1 (dep)

Obtain answer \( x = 3 \) only

A1

[4]

(iii) Attempt to form an equation in \( p \) e.g. by equating gradients of \( OP \) and the tangent at \( P \), or by substituting \((0, 0)\) in the equation of the tangent at \( P \)

M1

Obtain equation in any correct form e.g. \( \frac{4-p}{p} = 3 - p \)

A1

Obtain 3-term quadratic \( p^2 - 4p + 4 = 0 \), or equivalent

A1

Attempt to solve a quadratic equation in \( p \)

M1

Obtain answer \( p = 2 \) only

A1

[5]

7 (i) Attempt to differentiate using the quotient, product or chain rule

M1

Obtain derivative in any correct form

A1

Obtain the given answer correctly

A1

[3]
(ii) State or imply the indefinite integral is \(-\cot x\)  
Substitute limits and obtain given answer correctly  
\[ 2 \]

(iii) Use \(\cot^2 x = \csc^2 x - 1\) and attempt to integrate both terms,  
or equivalent  
Substitute limits where necessary and obtain a correct unsimplified answer  
Obtain final answer \(\sqrt{3} - \frac{1}{3} \pi\)  
\[ 3 \]

(iv) Use \(\cos 2A\) formula and reduce denominator to \(2\sin^2 x\)  
Use given result and obtain answer of the form \(k \sqrt{3}\)  
Obtain correct answer \(\frac{1}{2} \sqrt{3}\)  
\[ 3 \]
1 **EITHER:** State or imply non-modular inequality \(-5 < 2^x - 8 < 5\), or \((2^x - 8)^2 < 5^2\) or corresponding pair of linear equations or quadratic equation

B1

Use correct method for solving an equation of the form \(2^x = a\)

M1

Obtain critical values 1.58 and 3.70, or exact equivalents

A1

State correct answer 1.58 < \(x\) < 3.70

A1

**OR:** Use correct method for solving an equation of the form \(2^x = a\)

M1

Obtain one critical value (probably 3.70), or exact equivalent

A1

Obtain the other critical value, or exact equivalent

A1

State correct answer 1.58 < \(x\) < 3.70

A1

[4]

[Allow 1.59 and 3.7. Condone \(\leq\) for <. Allow final answers given separately. Exact equivalents must be in terms of ln or logarithms to base 10.]

[SR: Solutions given as logarithms to base 2 can only earn M1 and B1 of the first scheme.]

2 **EITHER:** Obtain correct unsimplified version of the \(x^2\) or \(x^4\) term of the expansion of

\((1 + \frac{1}{2}x^2)^{-2}\) or \((2 + x^2)^{-2}\)

M1

State correct first term \(\frac{1}{4}\)

B1

Obtain next two terms \(-\frac{1}{4}x^2 + \frac{1}{16}x^4\)

A1+A1

[The M mark is not earned by versions with unexpanded binomial coefficients such as \(\left(\frac{-2}{4}\right)\).]

[SR: Answers given as \(\frac{1}{4}(1 - x^2 + \frac{1}{4}x^4)\) earn M1B1A1.]

[SR: Solutions involving \(k(1 + \frac{1}{2}x^2)^{-2}\), where \(k = 2, 4\) or \(\frac{1}{4}\) can earn M1 and A1 for a correct simplified term in \(x^2\) or \(x^4\).]

**OR:** Differentiate expression and evaluate \(f(0)\) and \(f'(0)\), where \(f'(x) = kx(2 + x^2)^{-3}\)

M1

State correct first term \(\frac{1}{4}\)

B1

Obtain next two terms \(-\frac{1}{4}x^2 + \frac{1}{16}x^4\)

A1+A1

[Allow exact decimal equivalents as coefficients.]

[4]

3 Use correct \(\cos 2A\) formula, or equivalent pair of correct formulas, to obtain an equation in \(\cos \theta\)

M1

Obtain 3-term quadratic \(6 \cos^2 \theta + \cos \theta - 5 = 0\), or equivalent

A1

Attempt to solve quadratic and reach \(\theta = \cos^{-1}(a)\)

M1

Obtain answer 33.6\(^\circ\) (or 33.5\(^\circ\)) or 0.586 (or 0.585) radians

A1

Obtain answer 180\(^\circ\) or \(\pi\) (or 3.14) radians and no others in range

A1

[The answer \(\theta = 180^\circ\) found by inspection can earn B1.]

[Ignore answers outside the given range.]

[5]
4(i) EITHER Obtain terms $\frac{1}{2\sqrt{x}}$ and $\frac{1}{2\sqrt{y}} \frac{dy}{dx}$, or equivalent

Obtain answer in any correct form, e.g. $\frac{dy}{dx} = \frac{y}{\sqrt{x}}$ B1

OR: Using chain or product rule, differentiate $(\sqrt{a} - \sqrt{x})^2$ M1
Obtain derivative in any correct form A1
Express $\frac{dy}{dx}$ in terms of $x$ and $y$ only in any correct form A1

OR: Expand $(\sqrt{a} - \sqrt{x})^2$, differentiate and obtain term $-2 \cdot \frac{\sqrt{a}}{2\sqrt{x}}$, or equivalent B1
Obtain term 1 by differentiating an expansion of the form $a + x \pm 2\sqrt{a}\sqrt{x}$ B1
Express $\frac{dy}{dx}$ in terms of $x$ and $y$ only in any correct form B1

[3]

(ii) State or imply coordinates of $P$ are $(\frac{1}{2}a, \frac{1}{2}a)$ B1
Form equation of the tangent at $P$ M1
Obtain 3 term answer $x + y = \frac{1}{2}a$ correctly, or equivalent A1

[3]

5 (i) Make recognizable sketch of $y = \sec x$ or $y = 3 - x^2$, for $0 < x < \frac{\pi}{2}$ B1
Sketch the other graph correctly and justify the given statement B1

[2]

[Award B1 for a sketch with positive $y$-intercept and correct concavity. A correct sketch of $y = \cos x$ can only earn B1 in the presence of $1/(3 - x^2)$. Allow a correct single graph and its intersection with $y = 0$ to earn full marks.]

(ii) State or imply equation $\alpha = \cos^{-1}(1/(3 - \alpha^2))$ or $\cos \alpha = 1/(3 - \alpha^2)$ B1
Rearrange this in the form given in part (i) i.e. $\sec \alpha = 3 - \alpha^2$ B1

[2]

[Or work vice versa.]

(iii) Use the iterative formula with $0 \leq x_1 \leq \sqrt{2}$ M1
Obtain final answer 1.03 A1
Show sufficient iterations to justify its accuracy to 2d.p. or show there is a sign change in the interval (1.025, 1.035) A1

[3]
6 (i) Use product or quotient rule to find derivative M1
Obtain derivative in any correct form A1
Equate derivative to zero and solve a linear equation in \( x \) M1
Obtain answer \( 3 \frac{1}{2} \) only A1

(ii) State first step of the form \( \pm \frac{1}{2}(3-x)e^{-2x} \pm \frac{1}{2} \int e^{-2x} \, dx \), with or without 3 M1
State correct first step e.g. \( -\frac{1}{2}(3-x)e^{-2x} - \frac{1}{2} \int e^{-2x} \, dx \), or equivalent, with or without 3 A1
Complete the integration correctly obtaining \( -\frac{1}{4}(3-x)e^{-2x} + \frac{1}{2}e^{-2x} \), or equivalent A1
Substitute limits \( x = 0 \) and \( x = 3 \) correctly in the complete integral M1
Obtain answer \( \frac{1}{4}(5+e^{-6}) \), or equivalent (allow \( e^{0} \) in place of 1) A1

[5]

7 (i) EITHER: Attempt multiplication of numerator and denominator by \( 3 + 2i \), or equivalent M1
Simplify denominator to 13 or numerator to 13 + 26i A1
Obtain answer \( u = 1 + 2i \) A1

OR: Using correct processes, find the modulus and argument of \( u \) M1
Obtain modulus \( \sqrt{5} \) (or 2.24) or argument \( \tan^{-1}2 \) (or 63.4° or 1.11 radians) A1
Obtain answer \( u = 1 + 2i \) A1

[3]

(ii) Show the point \( U \) on an Argand diagram in a relatively correct position B1 \( \sqrt{\text{ }} \)
Show a circle with centre \( U \) B1 \( \sqrt{\text{ }} \)
Show a circle with radius consistent with 2 B1 \( \sqrt{\text{ }} \)

[f.t. on the value of \( u \).]

(iii) State or imply relevance of the appropriate tangent from \( O \) to the circle B1 \( \sqrt{\text{ }} \)
Carry out a complete strategy for finding max \( \text{arg } z \) M1
Obtain final answer 126.9° (2.21 radians) A1

[3]

[Drawing the appropriate tangent is sufficient for B1 \( \sqrt{\text{ }} \).
[A final answer obtained by measurement earns M1 only.]
8 (i) EITHER: Divide by denominator and obtain a quadratic remainder
   Obtain \( A = 1 \) \( M1 \)
   \( B, C \) or \( D \) \( A1 \)
   Obtain one correct answer \( A1 \)
   Obtain \( B = -1, C = 2, D = 0 \) \( A1 \)

   OR: Reduce RHS to a single fraction and identify numerator with that of \( f(x) \)
   Obtain \( A = 1 \) \( A1 \)
   \( B, C \) or \( D \) \( M1 \)
   Obtain one correct answer \( A1 \)
   Obtain \( B = -1, C = 2, D = 0 \) \( A1 \)

   [5]

(ii) Integrate and obtain terms \( x - \ln(x - 1) \), or equivalent \( B1 \)
   Obtain third term \( \ln(x^2 + 1) \), or equivalent \( B1 \)
   Substitute correct limits correctly in the complete integral \( M1 \)
   Obtain given answer following full and exact working \( A1 \)

   [4]

   [If \( B = 0 \) the first \( B1 \) is not available.]
   [SR: If \( A \) is omitted in part (i), treat as if \( A = 0 \). Thus only M1M1 and B1\(\sqrt{B1}\)M1 are available.]

9 (i) Separate variables and attempt to integrate \( \frac{1}{\sqrt{(P - A)}} \) \( M1 \)
   Obtain term \( 2\sqrt{(P - A)} \) \( A1 \)
   Obtain term \(-kt\) \( A1 \)

   [3]

(ii) Use limits \( P = 5A, t = 0 \) and attempt to find constant \( c \) \( M1 \)
   Obtain \( c = 4\sqrt{A} \), or equivalent \( A1 \)
   Use limits \( P = 2A, t = 2 \) and attempt to find \( k \) \( M1 \)
   Obtain given answer \( k = \sqrt{A} \) correctly \( A1 \)

   [4]

(iii) Substitute \( P = A \) and attempt to calculate \( t \)
   Obtain answer \( t = 4 \) \( A1 \)

   [2]

(iv) Using answers to part (ii), attempt to rearrange solution to give \( P \) in terms of \( A \) and \( t \) \( M1 \)
   Obtain \( P = \frac{1}{4}A(4 + (4 - t)^2) \), or equivalent, having squared \( \sqrt{A} \) \( A1 \)

   [2]

   [For the M1, \( \sqrt{(P - A)} \) must be treated correctly.]
10 (i) Express general point of \( l \) or \( m \) in component form e.g. \((1 + 2s, s, -2 + 3s)\) or \((6 + t, -5 - 2t, 4 + t)\)  
Equate at least two corresponding pairs of components and attempt to solve for \( s \) or \( t \)  
Obtain \( s = 1 \) or \( t = -3 \)  
Verify that all three component equations are satisfied  
Obtain position vector \( 3i + j + k \) of intersection point, or equivalent  

(ii) EITHER: Use scalar product to obtain \( 2a + b + 3c = 0 \) and \( a - 2b + c = 0 \)  
Solve and find one ratio e.g. \( a : b \)  
State one correct ratio  
Obtain answer \( a : b : c = 7 : 1 : -5 \), or equivalent  
Substitute coordinates of a relevant point and values of \( a \), \( b \) and \( c \) in general equation of plane and calculate \( d \)  
Obtain answer \( 7x + y - 5z = 17 \), or equivalent  

OR: Using two points on \( l \) and one on \( m \) (or vice versa) state three simultaneous equations in \( a \), \( b \), \( c \) and \( d \) e.g. \( 3a + b + c = d \), \( a - 2c = d \) and \( 6a - 5b + 4c = d \)  
Solve and find one ratio e.g. \( a : b \)  
State one correct ratio  
Obtain a ratio of three unknowns e.g. \( a : b : c = 7 : 1 : -5 \), or equivalent  
Use coordinates of a relevant point and found ratio to find fourth unknown e.g. \( d \)  
Obtain answer \( 7x + y - 5z = 17 \), or equivalent  

OR: Form a correct 2-parameter equation for the plane,  
e.g. \( \mathbf{r} = i - 2k + \lambda(2i + j + 3k) + \mu(i - 2j + k) \)  
State 3 equations in \( x \), \( y \), \( z \), \( \lambda \) and \( \mu \)  
State 3 correct equations  
Eliminate \( \lambda \) and \( \mu \)  
Obtain equation in any correct unsimplified form  
Obtain \( 7x + y - 5z = 17 \), or equivalent  

OR: Attempt to calculate vector product of vectors parallel to \( l \) and \( m \)  
Obtain two correct components of the product  
Obtain correct product, e.g. \( 7i + j - 5z \)  
State that the plane has equation of the form \( 7x + y - 5z = d \)  
Substitute coordinates of a relevant point and calculate \( d \)  
Obtain answer \( 7x + y - 5z = 17 \), or equivalent  

[The follow through is on \( 3i + j + k \) only.]
November 2003

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/04

MATHEMATICS
Paper 4 (Mechanics 1)
1 (i) The force is 320 N

(ii) For using Newton’s second law (3 terms needed)

\[320 - R = 100 \times 0.5\]

Resistance is 270 N

2 (i) Speed is 20 ms\(^{-1}\)

(ii) For using 

\[45 = \frac{1}{2} 10r^2\]

Time taken is 3 s

(iii) For using 

\[v^2 = u^2 + 2gs\]

Distance fallen is 35 m

3 (i) For using the idea of work as a force times a distance 

\[(25 \times 2 \cos 15^\circ)\]

Work done is 48.3 J

(ii) For resolving forces vertically (3 terms needed)

\[N + 25 \sin 15^\circ = 3 \times 10\]

Component is 23.5 N

4 (i) KE (gain) = \(\frac{1}{2} 0.15 \times 8^2\)

For using PE loss = KE gain

Height is 3.2 m

(ii) For using WD is difference in PE loss and KE gain

\[WD = 0.15 \times 10 \times 4 - \frac{1}{2} 0.15 \times 6^2\]

Work Done is 3.3 J

SR For candidates who treat \(AB\) as if it is straight and vertical (implicitly or otherwise) Max 2 out of 6 marks.

(i) \[s = 8^2 \div (2 \times 10) = 3.2\]  B1

(ii) \[a = 6^2 \div (2 \times 4) = 4.5\] and \[R = 0.15 \times 10 - 0.15 \times 4.5 = 0.825\] and 

\[WD = 4 \times 0.825 = 3.3\]  B1
5 (i) For applying Newton’s second law to $A$ or to $B$ (3 terms needed)  

\[
T - 0.6 = 0.4a \text{ or } 0.1g - T = 0.1a
\]

M1

For a second of the above 2 equations or for

\[
0.1g - 0.6 = 0.5a \quad [Can \ be \ scored \ in \ part \ (ii)]
\]

B1

(Sign of $a$ must be consistent with that in first equation)

Tension is 0.92 N  

A1 4

(ii) \[ a = 0.8 \]

B1

For using $v = at$

M1

Speed = 1.2 ms$^{-1}$  

A1 3

6 (i) \[ T_{BM} = T_{AM} \text{ or } T_{BM} \cos 30^\circ = T_{AM} \cos 30^\circ \]

B1

For resolving forces at $M$ horizontally ($2T \sin 30^\circ = 5$) or for using the sine rule in the triangle of forces ($T \div \sin 60^\circ = 5 \div \sin 60^\circ$) or for using Lami’s theorem ($T \div \sin 120^\circ = 5 \div \sin 120^\circ$)  

M1

Tension is 5 N  

A1 3

(ii) For resolving forces on $B$ horizontally ($N = T \sin 30^\circ$) or from symmetry ($N = 5/2$) or for using Lami’s theorem ($N \div \sin 150^\circ = 5 \div \sin 90^\circ$)  

M1

For resolving forces on $B$ vertically (3 terms needed) or for using Lami’s theorem

\[
0.2 \times 10 + F = T \cos 30^\circ \text{ or } (0.2g + F) \div \sin 120^\circ = T \div \sin 90^\circ
\]

A1

For using $F = \mu R$  

(2.33 = 2.5$\mu$)  

M1

Coefficient is 0.932  

A1 5

(iii) \[ (0.2 + m)g - 2.33 = 5 \cos 30^\circ \text{ or } mg = 2(2.33) \]

B1 \sqrt{}

\[
m = 0.466
\]

B1 2

7 (i) For using the idea that area represents the distance travelled.

M1

For any two of $\frac{1}{2} \times 100 \times 4.8$, $\frac{1}{2} \times 200(4.8 + 7.2)$, $\frac{1}{2} \times 200 \times 7.2$  

(240, 1200, 720)  

A1

Distance is 2160 m  

A1 3

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(ii) For using the idea that the initial acceleration is the gradient of
the first line segment or for using $v = at$ ($4.8 = 100a$) or
$v^2 = 2as$ ($4.8^2 = 2a \times 240$)  \hspace{1cm} M1
Accelaration is 0.048 ms$^{-2}$  \hspace{1cm} A1  \hspace{1cm} 2

(iii) $a = 0.06 - 0.00024t$  \hspace{1cm} B1

Acceleration is greater by 0.012 ms$^{-2}$ \[
\sqrt{v \text{ for } 0.06 - \text{ans(ii)}} \\
\text{(must be +ve) and/or wrong coefficient of } t \text{ in } a(t)] \\
[\text{Accept ‘acceleration is 1.25 times greater’}]  \hspace{1cm} B1 \sqrt{\phantom{0}}  \hspace{1cm} 2

(iv) $B$’s velocity is a maximum when $0.06 - 0.00024t = 0$ \hspace{1cm} B1 \sqrt{\phantom{0}}

[\sqrt{\text{wrong coefficient of } t \text{ in } a(t)}]

For the method of finding the area representing $s_B$ ($250$) \hspace{1cm} M1

\[
240 + \frac{1}{2} (4.8 + 6.6)150 \hspace{1cm} \text{or} \\
240 + (4.8 \times 150 + \frac{1}{2} 0.012 \times 150^2) \hspace{1cm} (1095) \hspace{1cm} A1
\]

For using the idea that $s_B$ is obtained from integration \hspace{1cm} M1

$0.03t^2 - 0.00004t^3$  \hspace{1cm} A1

Required distance is 155 m \hspace{1cm} A1 \sqrt{\phantom{0}}  \hspace{1cm} 6

(\sqrt{\text{dependent on both M marks}})
# MARK SCHEME

**MAXIMUM MARK: 50**

**SYLLABUS/COMPONENT: 9709/05, 8719/05**

MATHEMATICS AND HIGHER MATHEMATICS
Paper 5 (Mechanics 2)
1. For using Newton’s second law with \( a = \frac{v^2}{r} \)
   \[
   F = 50000 \times \frac{25^2}{1250}
   \]
   Magnitude of the force is 25 000 N

2. (i) For stating or implying that the centre of mass is vertically above the lowest point of the cone, and with \( y = 5 \)
   \[
   \text{For using } \tan \theta = \frac{10}{y} \text{ or equivalent}
   \]
   \( \theta = 63.4^\circ \)

(ii) For using \( F < \mu R \)
   \[
   mg \sin \theta < \mu mg \cos \theta
   \]
   Alternative for the above 2 marks:
   For using \( \mu = \tan \phi \) where \( \phi \) is the angle of friction
   \( \phi > \theta \) because cone topples without sliding
   Coefficient is greater than 2 (ft on tan\( \theta \) in (i))
   N.B. Direct quotation of “topples if \( \mu > \tan \theta \)” (scores B2); \( \mu > 2 \) (B1)

3. (i) \( T = \frac{88 \times 0.1}{0.4} \)
   For using Newton’s second law \( (22 - 0.2 \times 10 = 0.2a) \)
   (3 term equation needed)
   Initial acceleration is 100 ms\(^{-2} \)

(ii) For using \( E_{PE} = \frac{2x^2}{2L} \)
    \[
    (\frac{88 \times 0.1^2}{2 \times 0.4})
    \]
    Initial elastic energy is 1.1 J

(iii) Change in GPE = 0.2 \times 10 \times 0.1
   For using the principle of conservation of energy (KE, EPE and GPE must all be represented)
   \[
   \frac{1}{2} 0.2v^2 = 1.1 - 0.2
   \]
   Speed is 3 m s\(^{-1} \)
4  (i)  e.g. For taking moments about $BC$  

Distance of centre of mass of triangular portion is  

$9.5 + \frac{1}{3} \times 6 \quad (= 11.5)$  

$8 \times 9.5 \times 4.75 + \frac{1}{2} \times 8 \times 6 \times 11.5 = (8 \times 9.5 + \frac{1}{2} \times 8 \times 6) \bar{x}$  

Distance is 6.37 cm  

N.B.  Alternative method  

e.g. Moments about axis through $A$ perpendicular to $AB$  

Distance of C.O.M. of triangular piece removed is 2  

$(8 \times 15.5) \times 7.75 - (\frac{1}{2} \times 8 \times 6) \times 2 = (124 - 20) \bar{x_1}$  

$(\bar{x_1} = 9.13)$ therefore distance is 6.37 cm  

[4]  

(ii) For taking moments about $A$  

For LHS of $80(15.5 - 6.37) = T \times 15.5 \sin 30^\circ$  

For RHS of above equation  

Tension is 94.2 N  

[4]  

(iii) For resolving forces on the lamina vertically (3 term equation)  

$(V = 80 - 94.2 \times 0.5)$ or taking moments about $B$  

$(15.5V = 8 \times 10 \times 6.37)$  

Magnitude of vertical component is 32.9 N  

[2]
5 (i) For using $\dot{y} = \dot{y}_0 - gt$ with $\dot{y} = 0 \quad (t = 2\sin \alpha)$  
For using $y = \dot{y}_0 t - \frac{1}{2} gt^2$ with $t$ as found and $y = 7.2$, or show $t = 1.2$ as in (ii)  
Alternatively for using $y_{\text{max}} = \frac{V^2 \sin^2 \alpha}{2g}$ with $y_{\text{max}} = 7.2$ and $V = 20$  
or $\dot{y}^2 = \dot{y}_0^2 - 2gy$ with $\dot{y} = 0$  
$7.2 = \frac{400\sin^2 \alpha}{20}$  
Angle is 36.9° [4]

(ii) Speed on hitting the wall is $20 \times 0.8$  
(use of ball rebounding at 10 ms$^{-1}$ scores B0)  
For using $y = 0 - \frac{1}{2} gt^2 \quad (-7.2 = - \frac{1}{2} \times 10 t^2)$ or  
$0 = \dot{y} - gt \quad (0 = 12 - 10t)$  
$t = 1.2$  
Distance is 9.6 m (No ft if rebound velocity = 10 ms$^{-1}$)  
Alternative – speed on hitting the wall is $20 \times 0.8$  
Use trajectory equation, with $\theta = 0°$  
$-7.2 = x \tan 0° - \frac{gx^2}{2.8^2 \cos^2 0°}$  
$\allowe ft with halving attempt including 10$  
$x = 9.6$ m [4]

(iii) $\dot{y} = \mp 10 \times 1.2$  
$\theta = \tan^{-1} \left( \mp \frac{\dot{y}}{\dot{x}} \right)$  
($\dot{x}$ must have halving attempt. Allow $\dot{x} = 10$)  
Required angle is 56.3° [3]
6  (i) For using Newton’s second law

\[ 120 - 8v - 80 \times 10 \times 0.1 = 80a \]

\[ \frac{1}{5-v} \frac{dv}{dt} = \frac{1}{10} \text{ from correct working} \]

M1

A1

A1

[3]

(ii) For separating the variables and attempting to integrate

\[ -\ln(5 - v) = \frac{1}{10} t + (C) \]

For using \( v(0) = 0 \) to find \( C \) (or equivalent by using limits)

\( C = -\ln5 \)

M1

For converting the equation from logarithmic to exponential form

(allow even if + \( C \) omitted) \( 5 \div (5 - v) = e^{t/10} \)

\[ v = 5(1 - e^{-t/10}) \text{ from correct working} \]

M1

A1

[5]

(iii) For using \( v = \frac{dx}{dt} \) and attempting to integrate

\[ x = 5(t + 10e^{-t/10}) + (C) \]

A1 ft

For using \( x(0) = 0 \) to find \( C \) (= −50), then substituting \( t = 20 \)

(OR equivalent using limits)

Length is 56.8 m

A1

OR

For using Newton’s second law with \( a = \frac{dv}{dx} \), separating the variables and attempting to integrate

\[ -v - 5\ln(5 - v) = \frac{x}{10} + C \]

M1

A1

For using \( v = 0 \) when \( x = 0 \) to find \( C \) (= −5ln5), then substituting \( t = 20 \) into \( v(t) \)

\( v(20) = 5(1 - e^{-2}) = 4.3233 \),

And finally substituting \( v(20) \) into the above equation

\( x = -50(1 - e^{-2}) + 50 \times 2 = 50 + 50e^{-2} \)

M1

Length is 56.8 m

A1

[4]
### 1

<table>
<thead>
<tr>
<th>x</th>
<th>freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>23</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
</tr>
</tbody>
</table>

**OR**

- \( P(0) = \frac{23}{40} \), \( P(2) = \frac{17}{40} \)
- Mean = \( \frac{34}{40} = 0.850 \)
- Variance = \( \frac{(4\times17) - \left(0.85\right)^2}{40} = 0.978 \) (exact answer 0.9775) \( \left(\frac{391}{400}\right) \)

**For reasonable attempt at the mean using freqs or probs but not using prob=0.5**

**For correct mean**

**For correct variance formula**

**For correct answer**

### 2

- frequencies: 3, 7, 6, 3, 1
- scaled frequencies: 3, 7, 3, 1.5, 0.5
  - or 0.006, 0.014, 0.006, 0.003, 0.001

**For frequencies and attempt at scaling, accept cw/freq but not cw \times freq, not cw/mid point**

**For correct heights from their scaled frequencies seen on the graph**

**For correct widths of bars, uniform horiz scale, no halves or gaps or less-than-or-equal tos**

**Both axes labelled, fd and area or m². Not class width**

### 3

- \( 28 - \mu = 0.496\sigma \) (accept 0.495 or in between)
- \( 35 - \mu = 1.282\sigma \) (accept 1.281 or in between, but not 1.28)
- \( \sigma = 8.91 \) (accept 8.89 to 8.92 incl)
- \( \mu = 23.6 \)

**For any equation with \( \mu \) and \( \sigma \) and a reasonable \( z \) value not a prob. Allow cc, \( \sqrt{\sigma} \), \( \sigma^2 \), or – and give M1 A0A1ft for these four cases**

**For 2 correct equations**

**For solving their two equations by elim 1 variable sensibly**

**For correct answer**

### 4

- (i) \( (0.95)^5 = 0.774 \)
- (ii) \( (0.95)^4 \times (0.05)^1 \times \binom{5}{1} \)
  \[= 0.204 \]
- (iii) \( (0.95)^2 \times (0.05) \)
  \[= 0.0451 (361/8000) \]

**For 0.95 seen, can be implied**

**For correct final answer**

**For any binomial calculation with 3 terms, powers summing to 5**

**For correct answer**

**For no Ps, no Cs, and only 3 terms of type \( p^2 (1-p) \)**

**For correct answer**
5

\[ P(M|C) = \frac{0.54 \times 0.05}{0.54 \times 0.05 + 0.46 \times 0.02} \]
\[ = 0.746 \ (135/181) \]

For correct shape ie \( M \) and \( F \) first

All correct, ie labels and probabilities, no labels gets M1 only for (implied)correct shape

6 (a) (i) 18564

\( \begin{align*}
\text{or } 6/18 \times \text{their (i) or } 18C_6 - 17C_6
\end{align*} \)
\[ = 6188 \]

For finding \( P(M \text{ and } C) \) and \( P(F \text{ and } C) \)

M1 For using 4 correct probs

For correct conditional probability

B1 For correct numerator

M1 For summing two two-factor ‘terms’

A1 6 For correct answer

(b) (i) 40320

\[ \begin{align*}
5! \times 3! \times 4C_1
\end{align*} \]
\[ = 2880 \]

For correct final answer

M1 For using 17 and 5 as a perm or comb

A1 2 For correct answer

(ii) \( 5! = 5 \times 4! \)

B1 1 For correct final answer

B1 For 5! or \( P \) used in a prod or quotient with a term \( \neq 5! \)

B1 For 3!

B1 For \( 4C_1 \), may be implied by 4!

B1 4 For correct final answer

7 (i) \( z = \pm 1.143 \)

\[ \begin{align*}
P(7.8<T<11) &= \Phi(1.143) - 0.5 \\
&= 0.8735 - 0.5 \\
&= 0.3735 \ (\text{accept ans rounded to 0.37 to } 0.374) \\
\end{align*} \]

For standardising, can be implied, no cc, no \( \sigma^2 \) but accept \( \sqrt{\sigma} \)

A1 For seeing 0.8735

M1 For subtracting two probs, \( p_2 - p_1 \) where \( p_2 > p_1 \)

A1 4 For correct answer

(ii) \( (0.1265)^2 \times (0.8735) \times 3C_2 \)

\[ \begin{align*}
&= 0.0419
\end{align*} \]

For any three term binomial-type expression with powers summing to 3

A1ft 2 For correct answer ft on their 0.8735/0.1265

(iii) Not symmetric so not normal

Does not agree with the hospital’s figures

B1 For any valid reason

B1dep 2 For stating it does not agree, with no invalid reasons

8 (i) 18c = 1

\[ c = 1/18 = 0.0556 \]

M1 For \( \sum p_i = 1 \)

A1 2 For correct answer

(ii) \( E(X) = 2.78 \ (25/9)(= 50c) \)

\[ \begin{align*}
\text{Var } (X) &= 1.17 \ (95/81)(=160c - 2500 \ c^2)
\end{align*} \]

M1 Using correct formula for \( E(X) \)

A1ft For correct expectation, ft on their c

M1 For correct variance formula

A1ft 4 For correct answer ft on their c

(iii) \( P(X > 2.78) = 11c \)

\[ 0.611 \ (11/18) \]

M1 For using their correct number of discrete values of \( X \)

A1 2 For correct answer
<table>
<thead>
<tr>
<th>Q</th>
<th>Expression</th>
<th>Mark Scheme</th>
<th>Syllabus</th>
<th>Paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[ \sqrt{n} \times 1.96 &lt; 1 ] &lt;br&gt; [ n &gt; 13.9 \ (13.87) ] &lt;br&gt; [ n = 14 ]</td>
<td>M1 A1 M1 A1 [4]</td>
<td>For equality or inequality involving width or equivalent and term in (1/\sqrt{n}) and a (z)-value&lt;br&gt;For correct inequality&lt;br&gt;For solving a relevant equation&lt;br&gt;For correct answer cwo</td>
<td>9709/8719 7</td>
</tr>
<tr>
<td>2</td>
<td>( \lambda = 4.5 )</td>
<td>M1 B1 M1 A1 [5]</td>
<td>For using Poisson approximation any mean&lt;br&gt;For correct mean used&lt;br&gt;For calculating (P(2, 3, 4)) their mean&lt;br&gt;For correct numerical expression&lt;br&gt;For correct answer&lt;br&gt;NB Use of Normal can score B1 M1&lt;br&gt;SR Correct Bin scores M1 A1 A1 only</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( SU \sim N(19,12) ) &lt;br&gt; ( P( T-SU &gt; 0) ) or ( P(T-S &gt; 5) = 1 - \Phi \left( \frac{0-1}{\sqrt{21}} \right) ) &lt;br&gt; [ = 0.586 ]</td>
<td>B1 M1 M1 M1 A1 [5]</td>
<td>For correct mean and variance. Can be implied if using (P(T-S&gt;5)) in next part&lt;br&gt;For consideration of (P(T-SU &gt; 0))&lt;br&gt;For summing their two variances&lt;br&gt;For normalising and finding correct area from their values&lt;br&gt;For correct answer</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(i) [ \lambda = \frac{20}{80} = 0.25 ] &lt;br&gt; [ P(X \geq 3) = 1 - P(X \leq 2) ] &lt;br&gt; [ = 1 - e^{-0.25} \left( 1 + 0.25 + \frac{0.25^2}{2} \right) ] &lt;br&gt; [ = 0.00216 ]</td>
<td>B1 M1 M1 A1 [4]</td>
<td>For ( \lambda = 0.25 )&lt;br&gt;For calculating a relevant Poisson prob( any ( \lambda ))&lt;br&gt;For calculating expression for (P(X \geq 3)) their ( \lambda )&lt;br&gt;For correct answer</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(ii) [ \frac{-k}{e^{80}} = 0.9 ] &lt;br&gt; [ \frac{-k}{80} = -0.10536 ] &lt;br&gt; [ k = 8.43 ]</td>
<td>M1 M1 M1 A1 [4]</td>
<td>For using ( \lambda = -t/80 ) in an expression for (P(0))&lt;br&gt;For equating their expression to 0.9&lt;br&gt;For solving the associated equation&lt;br&gt;For correct answer cwo</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>(i) [ P( \bar{X} &gt; 1800) = 1 - \Phi \left( \frac{1800 - 1850}{117 / \sqrt{26}} \right) ] &lt;br&gt; [ = 0.985 ]</td>
<td>B1 M1 A1 [3]</td>
<td>For 117/(\sqrt{26}) (or equiv)&lt;br&gt;For standardising and use of tables&lt;br&gt;For correct answer cwo</td>
<td></td>
</tr>
</tbody>
</table>
(ii) $H_0: \mu = 1850$
$H_1: \mu \neq 1850$

Test statistic $= \frac{1833 - 1850}{117/\sqrt{26}} = -0.7409$

Critical value $z = \pm 1.645$

Accept $H_0$, no significant change

**Marks**

<table>
<thead>
<tr>
<th>Statement</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both hypotheses correct</td>
<td>B1</td>
</tr>
<tr>
<td>Standardising attempt including standard error</td>
<td>M1</td>
</tr>
<tr>
<td>Correct test statistic ($+/$-)$</td>
<td>A1</td>
</tr>
<tr>
<td>Comparing with $z = \pm 1.645$, $+ \text{ with } + \text{ or } - \text{ with } -$ (or equiv area comparison) $\text{ft } 1 \text{ tail test } z=1.282$</td>
<td>M1</td>
</tr>
<tr>
<td>For correct conclusion on their test statistic and their $z$. No contradictions.</td>
<td>A1ft</td>
</tr>
</tbody>
</table>

6 (i) (a) Rejecting $H_0$ when it is true
(b) Accepting $H_0$ when it is false

(ii) (a) $P(\text{NNNNN})$ under $H_0 = (0.94)^5 = 0.7339$
$P(\text{Type I error}) = 1 - 0.7339 = 0.266$

(b) $P(\text{NNNNN})$ under $H_1 = (0.7)^5 = 0.168$
$P(\text{Type II error}) = 0.168$

**Marks**

<table>
<thead>
<tr>
<th>Statement</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>For evaluating $P(\text{NNNNN})$ under $H_0$</td>
<td>M1*</td>
</tr>
<tr>
<td>For correct answer (could be implied)</td>
<td>A1</td>
</tr>
<tr>
<td>For identifying the Type I error outcome</td>
<td>M1*</td>
</tr>
<tr>
<td>For correct final answer</td>
<td>A1ft</td>
</tr>
<tr>
<td>SR If M0M0 allow B1 for Bin(5,0.94)used</td>
<td>dep*</td>
</tr>
<tr>
<td>Or equivalent</td>
<td></td>
</tr>
<tr>
<td>For Bin(5,0.7) used</td>
<td>M1</td>
</tr>
<tr>
<td>For $P(\text{NNNNN})$ under $H_1$</td>
<td>M1</td>
</tr>
<tr>
<td>For correct final answer</td>
<td>A1</td>
</tr>
</tbody>
</table>

7 (i) $\int_{0}^{\infty} ke^{-3x} \, dx = 1$

$0 - \frac{-k}{3} = 1 \Rightarrow k = 3$

(ii) $\int_{0}^{q1} 3e^{-3x} \, dx = 0.25$

$\left[ -e^{-3x} \right]_{0}^{q1} = 0.25$
$-e^{-3q1} + 1 = 0.25$
$0.75 = e^{3q1}$
$q_1 = 0.0959$

**Marks**

<table>
<thead>
<tr>
<th>Statement</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>For attempting to integrate from 0 to $\infty$ and putting the integral = 1</td>
<td>M1</td>
</tr>
<tr>
<td>For obtaining given answer correctly</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>[2]</td>
</tr>
<tr>
<td>For equating $\int 3e^{-3x} , dx$ to 0.25 (no limits needed)</td>
<td>M1</td>
</tr>
<tr>
<td>For attempting to integrate and substituting (sensible) limits and rearranging</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>[3]</td>
</tr>
</tbody>
</table>
(iii) Mean = \[\int_{0}^{\infty} 3xe^{-3x} \, dx\]

\[
= \left[ -xe^{-3x} \right]_{0}^{\infty} - \int_{0}^{\infty} e^{-3x} \, dx
\]

\[
= \left[ \frac{e^{-3x}}{-3} \right]_{0}^{\infty}
\]

\[
= 0.333 \text{ or } 1/3
\]

<table>
<thead>
<tr>
<th>Mark</th>
<th>Scheme</th>
<th>Syllabus</th>
<th>Paper</th>
</tr>
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<tbody>
<tr>
<td>B1</td>
<td></td>
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<tr>
<td>M1</td>
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<td></td>
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<tr>
<td>A1</td>
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</table>

For correct statement for mean
For attempting to integrate $3xe^{-3x}$ (no limits needed)
For $-xe^{-3x}$ or $-xe^{-3x}/3$
For attempt $\int_{0}^{\infty} e^{-3x} \, dx$ (their integral)
For $0+ \left[ \frac{e^{-3x}}{-3} \right]_{0}$
For correct answer

[6]