Mark Scheme Notes

- Marks are of the following three types:

  **M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  **A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  **B** Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0. B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking $g$ equal to 9.8 or 9.81 instead of 10.
• The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)
AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO Correct Working Only – often written by a ‘fortuitous' answer
ISW Ignore Subsequent Working
MR Misread
PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS See Other Solution (the candidate makes a better attempt at the same question)
SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

• **MR -1** A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √"marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

• **PA -1** This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.
1. \((2x - 1/x)^5\). 4\(^{th}\) term needed.
   \[\rightarrow \binom{5}{3} = 10\]
   \[\rightarrow x^2 \times (-1)^3\]
   \[\rightarrow -40\]
   Must be 4\(^{th}\) term – needs \((2x)^2 (1/x)^3\)
   Includes and converts \(\binom{2}{3}\) or \(\binom{3}{3}\)
   Whole series given and correct term not quoted, allow 2/3

2. \(\sin 3x + 2 \cos 3x = 0\)
   \(\tan 3x = -2\)
   \(x = 38.9\) (8)
   and \(x = 98.9\) (8)
   and \(x = 158.9\) (8)
   Use of \(\tan = \sin \div \cos\) with \(3x\)
   For 60 + “his”
   For 120 + “his” and no others in range
   (ignore excess ans. outside range)
   Loses last A mark if excess answers in the range

3. (a) \(\frac{dy}{dx} = 4 - 12x^3\)
   (b) \(\int (2x^2 - 6x^{-1} + c)\)
   \((a)\) (quotient OK M1 correct formula, A1 co)
   \((b)\) One off for each error (4, -, 12, -3)
   \(3 \times B1\)
   One for each term – only give +c if obvious attempt at integration

4. \(a = -10\) \(a + 14d = 11\) \(d = \frac{3}{2}\)
   \(a + (n - 1)d = 41\) \(n = 35\)
   Either \(S_n = n/2(2a + (n - 1)d)\) or \(n/2(a + l)\)
   \(= 542.5\)
   Using \(a = (n - 1)d\)
   Correct method – not for \(a + nd\)
   Either of these used correctly
   For his \(d\) and any \(n\)

5. (i) \(2a + b = 1\) and \(5a + b = 7\)
   \(\rightarrow a = 2\) and \(b = -3\)
   Realising how one of these is formed
   Co
   (ii) \(f(x) = 2x - 3\) \(ff(x) = 2(2x - 3) - 3\)
   \(\rightarrow 4x - 9\)
   \(= 0\) when \(x = 2.25\)
   Replacing “x” by “his ax + b” and “+b”
   For his \(a\) and \(b\) and solved = 0
   Co
### 6. (i)
\[x = \frac{\pi}{2}, \quad y = 3\] (allow if 90°)
\[k = \frac{6}{\pi}\] co.

(ii) \((-\frac{\pi}{2}, -3)\) – must be radians

(iii) For complete cycle, shape including curves, not lines, -3 to +3 shown or implied, for \(-\pi\) to \(\pi\). Degrees ok

\[\text{B2, 1 [2]}\]

Realising maximum is \((\pi/2, 3)\) + sub Co (even if no graph)

\[\text{M1 A1 [2]}\]

Co (could come from incorrect graph)

\[\text{B1 [1]}\]

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### 7. (i)

Gradient of \(L_1\) = -2
Gradient of \(L_2\) = \(\frac{1}{2}\)
Equation of \(L_2\): \(y - 4 = \frac{1}{2}(x - 7)\)

(ii) Sim Eqns
\[x = 3, \quad y = 2\]

\[\text{AB} = \sqrt{2^2 + 4^2} = \sqrt{20}\] or 4.47

\[\text{B1 M1 M1A1 [4]}\]

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### 8. (i) \[\overrightarrow{BA} = \mathbf{a} - \mathbf{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}\]
\[\overrightarrow{BC} = \mathbf{c} - \mathbf{b} = -2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}\]

Dot product \(-2 + 8 - 6 = 0\)

→ Perpendicular

\[\text{M1 A1 [4]}\]

Knowing how to use position vector for \(\overrightarrow{BA}\) or \(\overrightarrow{BC}\) – not for \(\overrightarrow{AB}\) or \(\overrightarrow{CB}\)

Knowing how to use \(x_1y_1 + x_2y_2 + x_3y_3\).

Correct deduction. Beware fortuitous (uses \(\overrightarrow{AB}\) or \(\overrightarrow{CB}\) – can get 3 out of 4)

\[\text{M1A1 [4]}\]

Knowing how to get one of these

\[\text{M1 [4]}\]

Both correct + conclusion. Could be dot product \(= 60\) → angle \(= 0^\circ\)

\[\text{M1A1 [4]}\]

Knowing what to do. Co. Allow 5:2
9. (i) θ = 1 angle BOC = π - θ
Area = ½r²θ = 68.5 or 32(π-1)
(or ½circle-sector)

(ii) 8 + 8 + 8θ = ½(8 + 8 + 8(π-θ))
Solution of this eqn
→ 0.381 or ½(π-2)

(iii) θ = π/3
AB = 8cm
BC = 2 x 8sinπ/3 = 8√3
Perimeter = 24 + 8√3

For π-θ or for ½πr² – sector
M1 Use of ½πr²
A1 Co
NB. 32 gets M1 only

Relevant use of s = rθ twice
M1 Needs θ – collected – needs perimeters
A1 Co.

Valid method for BC – cos rule, Pyth
A1 Everything OK. Answer given
NB. Decimal check loses this mark

10. y = √(5x + 4)

(i) dy/dx = ½(5x + 4)⁻¹/₂ x 5
x = 1, dy/dx = 5/6
B1B1 B1 [3] ½(5x + 4)⁻¹/₂ x 5 B1 for each part
Co

(ii) dy/dt = dy/dx x dx/dt
= 5/6 x 0.03
→ 0.025
A1√ [2] For (i) x 0.03
M1 Chain rule correctly used

(iii) realises that area → integration

\[ \int = (5x + 4)^{3/2} + \frac{3}{2} - \frac{5}{5} \]

A1A1 For (5x + 4)^{3/2} + 3/2. For ÷ 5

Use of limits → 54/15 - 16/15
= 38/15 = 2.53
DM1 A1 [5] Must use “0” to “1”
11. (i) \[ 8x - x^2 = a - x^2 - b^2 - 2bx + \text{equating} \]
\[ \rightarrow b = -4 \]
\[ a = b^2 = 16 \text{ (i.e. } 16 - (x - 4)^2) \]

(ii) \[ \frac{dy}{dx} = 8 - 2x = 0 \text{ when} \]
\[ \rightarrow (4, 16) \text{ (or from } -b \text{ and } a) \]

(iii) \[ 8x - x^2 \geq -20 \]
\[ x^2 - 8x + 20 = (x - 10)(x + 2) \]
End values \(-2 \leq x \leq 10\)
Interval \(-2 \leq x \leq 10\)

\[ g: \ x \rightarrow 8x - x^2 \text{ for } x \geq 4 \]

(iv) domain of \( g^{-1} \) is \( x \leq 16 \)
range of \( g^{-1} \) is \( g^{-1} \geq 4 \)

(v) \[ y = 8x - x^2 \rightarrow x^2 - 8x + y = 0 \]
\[ x = 8 \pm \sqrt{(64 - 4y)} \div 2 \]
\[ g^{-1}(x) = 4 + \sqrt{(16 - x)} \]
\[ \text{or } (x - 4)^2 = 16 - y, \rightarrow x = 4 + \sqrt{(16 - y)} \]
\[ \rightarrow y = 4 + \sqrt{(16 - x)} \]

\[ \text{M1} \]
\[ \text{B1} \]
\[ \text{A1} \]
\[ [3] \]

\[ \text{Sets to } 0 + \text{ correct method of solution} \]
\[ \text{Co} - \text{ independent of } < \text{ or } > \text{ or } = \]
\[ \text{Co} - \text{ including } \leq (< \text{ gets } A0) \]

\[ \text{B1} \]
\[ \text{B1} \]
\[ [2] \]

\[ \text{From answer to (i) or (ii). Accept } <16 \]
\[ \text{Not f.t since domain of } g \text{ given} \]

\[ \text{M1} \]

\[ \text{DM1} \]
\[ \text{A1} \]
\[ [3] \]

\[ \text{Use of quadratic or completed square expression to make } x \text{ subject} \]
\[ \text{Replaces } y \text{ by } x \]
\[ \text{Co (inc. omission of -)} \]
1  EITHER: State or imply non-modular inequality \((x - 4)^2 > (x + 1)^2\), or corresponding equation B1
Expand and solve a linear inequality, or equivalent M1
Obtain critical value \(1\frac{1}{2}\) A1
State correct answer \(x < 1\frac{1}{2}\) (allow =) A1

OR: State a correct linear equation for the critical value e.g. \(4 - x = x + 1\) B1
Solve the linear equation for \(x\) M1
Obtain critical value \(1\frac{1}{2}\), or equivalent A1
State correct answer \(x < 1\frac{1}{2}\) A1

OR: State the critical value \(1\frac{1}{2}\), or equivalent, from a graphical method or by inspection or by solving a linear inequality B3
State correct answer \(x < 1\frac{1}{2}\) B1

[4]

2 (i) EITHER: Expand RHS and obtain at least one equation for \(a\) M1
Obtain \(a^2 = 9\) and \(2a = 6\), or equivalent A1
State answer \(a = 3\) only A1

OR: Attempt division by \(x^2 + ax + 1\) or \(x^2 - ax - 1\), and obtain an equation in \(a\) M1
Obtain \(a^2 = 9\) and either \(a^3 - 11a + 6 = 0\) or \(a^3 - 7a - 6 = 0\), or equivalent A1
State answer \(a = 3\) only A1

[Special case: the answer \(a = 3\), obtained by trial and error, or by inspection, or with no working earns B2.]

(ii) Substitute for \(a\) and attempt to find zeroes of one of the quadratic factors M1
Obtain one correct answer A1
State all four solutions \(\frac{1}{2}(-3 \pm \sqrt{5})\) and \(\frac{1}{2}(3 \pm \sqrt{13})\), or equivalent A1

[3]

3 (i) State or imply indefinite integral of \(e^{2x}\) is \(\frac{1}{2}e^{2x}\), or equivalent B1
Substitute correct limits correctly M1
Obtain answer \(R = \frac{1}{2} e^{2p} - \frac{1}{2}\), or equivalent A1

[3]

(ii) Substitute \(R = 5\) and use logarithmic method to obtain an equation in \(2p\) M1*
Solve for \(p\) M1 (dep*)
Obtain answer \(p = 1.2 (1.1989 ...\) A1

[3]
4 (i) Use tan \((A \pm B)\) formula to obtain an equation in \(\tan x\)

\[
\tan x + 1 = 4 \frac{1 - \tan x}{1 + \tan x}, \text{ or equivalent}
\]

Transform to a 2- or 3-term quadratic equation

Obtain given answer correctly

[4]

(ii) Solve the quadratic and calculate one angle, or establish that \(t = \frac{1}{3}, 3\) (only)

Obtain one answer, e.g. \(x = 18.4^\circ \pm 0.1^\circ\)

Obtain second answer \(x = 71.6^\circ\) and no others in the range

[Ignore answers outside the given range]

5 (i) Make recognizable sketch over the given range of two suitable graphs, e.g. \(y = \ln x\) and \(y = 2 - x^2\)

State or imply link between intersections and roots and justify given answer

[3]

(ii) Consider sign of \(\ln x - (2 - x^2)\) at \(x = 1\) and \(x = 1.4\), or equivalent

Complete the argument correctly with appropriate calculation

[2]

(iii) Use the given iterative formula correctly with \(1 \leq x_n \leq 1.4\)

Obtain final answer 1.31

Show sufficient iterations to justify its accuracy to 2d.p., or show there is a sign change in the interval (1.305, 1.315)

[3]

6 (i) Attempt to apply the chain or quotient rule

Obtain derivative of the form \(ksec^2 x\) or equivalent

Obtain correct derivative \(-sec^2 x\) or equivalent

Explain why derivative, and hence gradient of the curve, is always negative

[4]

(ii) State or imply correct ordinates: 1, 0.7071..., 0.5

Use correct formula, or equivalent, with \(h = \frac{1}{6}\pi\) and three ordinates

Obtain answer 0.57 (0.57220...) ± 0.01 (accept 0.18 \(\pi\))

[3]
(iii) Justify the statement that the rule gives an over-estimate \( B1 \) \[1\]

7 (i) State \( \frac{dx}{d\theta} = 2 - 2\cos 2\theta \) or \( \frac{dy}{d\theta} = 2\sin 2\theta \) \( B1 \)

Use \( \frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} \) \( M1 \)

Obtain answer \( \frac{dy}{dx} = \frac{2\sin 2\theta}{2 - 2\cos 2\theta} \) or equivalent \( A1 \)

Make relevant use of \( \sin 2A \) and \( \cos 2A \) formulae (indep.) \( M1 \)

Obtain given answer correctly \( A1 \)

\[5\]

(ii) Substitute \( \theta = \frac{\pi}{4} \) in \( \frac{dy}{dx} \) and both parametric equations \( M1 \)

Obtain \( \frac{dy}{dx} = 1, \ x = \frac{\pi}{2} - 1, \ y = 2 \) \( A1 \)

Obtain equation \( y = x + 1.43 \), or any exact equivalent \( A1 \sqrt{\} \)

\[3\]

(iii) State or imply that tangent is horizontal when \( \theta = \frac{\pi}{2} \) or \( \frac{3\pi}{2} \) \( B1 \)

Obtain a correct pair of \( x \), \( y \) or \( x \)- or \( y \)-coordinates \( B1 \)

State correct answers \( (\pi, 3) \) and \( (3\pi, 3) \) \( B1 \)

\[3\]
June 2003

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 75

SYLLABUS/COMPONENT: 9709/03, 8719/03

MATHEMATICS AND HIGHER MATHEMATICS
Paper 3 (Pure 3)
1 (i) Use trig formulae to express LHS in terms of sin x and cos x  

Use cos 60° = sin 30° to reduce equation to given form cos x = k  

M1  

[2]  

(ii) State or imply that \( k = \frac{1}{\sqrt{3}} \) (accept -0.577 or -0.58)  

A1  

Obtain answer \( x = 125.3° \) only  

A1  

[Answer must be in degrees; ignore answers outside the given range.]  

[SR: if \( k = \frac{1}{\sqrt{3}} \) is followed by \( x = 54.7° \), give A0A1.\]  

[2]  

2 State first step of the form \( kx e^{2x} \pm \int ke^{2x} \, dx \)  

M1  

Complete the first step correctly  

A1  

Substitute limits correctly having attempted the further integration of \( ke^{2x} \)  

M1  

Obtain answer \( \frac{1}{4} (e^2 + 1) \) or exact equivalent of the form \( ae^2 + b \), having used \( e^0 = 1 \) throughout  

A1  

[4]  

3 EITHER State or imply non-modular inequality \((x - 2)^2 < (3 - 2x)^2\), or corresponding equation  

B1  

Expand and make a reasonable solution attempt at a 2- or 3-term quadratic, or equivalent  

M1  

Obtain critical value \( x = 1 \)  

A1  

State answer \( x < 1 \) only  

A1  

OR State the relevant linear equation for a critical value,  

i.e. \( 2 - x = 3 - 2x \), or equivalent  

B1  

Obtain critical value \( x = 1 \)  

B1  

State answer \( x < 1 \)  

B1  

State or imply by omission that no other answer exists  

B1  

OR Obtain the critical value \( x = 1 \) from a graphical method, or by inspection,  

or by solving a linear inequality  

B2  

State answer \( x < 1 \)  

B1  

State or imply by omission that no other answer exists  

B1  

[4]
4 (i) EITHER State or imply that \( x - 2 \) is a factor of \( f(x) \) \( \text{B1} \)
Substitute 2 for \( x \) and equate to zero \( \text{M1} \)
Obtain answer \( a = 8 \) \( \text{A1} \)

[The statement \( (x - 2)^2 = x^2 - 4x + 4 \) earns \( \text{B1} \).]

OR Commence division by \( x^2 - 4x + 4 \) and obtain partial quotient \( x^2 + 2x \) \( \text{B1} \)
Complete the division and equate the remainder to zero \( \text{M1} \)
Obtain answer \( a = 8 \) \( \text{A1} \)

OR Commence inspection and obtain unknown factor \( x^2 + 2x + c \) \( \text{B1} \)
Obtain \( 4c = a \) and an equation in \( c \) \( \text{M1} \)
Obtain answer \( a = 8 \) \( \text{A1} \)

(ii) EITHER Substitute \( a = 8 \) and find other factor \( x^2 + 2x + 2 \) by inspection or division \( \text{B1} \)
State that \( x^2 - 4x + 4 \geq 0 \) for all \( x \) (condone \( > \) for \( \geq \)) \( \text{B1} \)
Attempt to establish sign of the other factor \( \text{M1} \)
Show that \( x^2 + 2x + 2 > 0 \) for all \( x \) and complete the proof \( \text{A1} \)
[An attempt to find the zeros of the other factor earns \( \text{M1} \).]

OR Equate derivative to zero and attempt to solve for \( x \) \( \text{M1} \)
Obtain \( x = -\frac{1}{2} \) and 2 \( \text{A1} \)
Show correctly that \( f(x) \) has a minimum at each of these values \( \text{A1} \)
Having also obtained and considered \( x = 0 \), complete the proof \( \text{A1} \)

5 (i) State or imply \( w = \cos \frac{2}{3} \pi + \sin \frac{2}{3} \pi \) (allow decimals) \( \text{B1} \)
Obtain answer \( uw = -\sqrt{3} - i \) (allow decimals) \( \text{B1} \)
Multiply numerator and denominator of \( \frac{u}{w} \) by \(-1 - i \sqrt{3} \), or equivalent \( \text{M1} \)
Obtain answer \( \frac{u}{w} = \sqrt{3} - i \) (allow decimals) \( \text{A1} \)

(ii) Show \( U \) on an Argand diagram correctly \( \text{B1} \)
Show \( A \) and \( B \) in relatively correct positions \( \text{B1} \)

(iii) Prove that \( AB = UA \) (or \( UB \)), or prove that angle \( AUB = angle ABU \) (or angle \( BAU \)) or prove, for example, that \( AO = OB \) and angle \( AOB = 120^\circ \), or prove that one angle of triangle \( UAB \) equals 60° \( \text{B1} \)
Complete a proof that triangle \( UAB \) is equilateral \( \text{B1} \)
6 (i) EITHER State or imply \( f(x) \equiv \frac{A}{2x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2} \)

State or obtain \( A = 1 \)
State or obtain \( C = 8 \)
Use any relevant method to find \( B \)
Obtain value \( B = 4 \)

OR State or imply \( f(x) \equiv \frac{A}{2x+1} + \frac{Dx + E}{(x-2)^2} \)

State or obtain \( A = 1 \)
Use any relevant method to find \( D \) or \( E \)
Obtain value \( D = 4 \)
Obtain value \( E = 0 \)

(ii) EITHER Use correct method to obtain the first two terms of the expansion of \( (1 + 2x)^{-1} \) or \( (x - 2)^{-1} \) or \( (x - 2)^2 \) or \( (1 - \frac{1}{2}x)^{-1} \) or \( (1 - \frac{1}{2}x)^{-2} \)

Obtain any correct sum of unsimplified expansions up to the terms in \( x^2 \) (deduct 1 mark for each incorrect expansion)
Obtain the given answer correctly

[Unexpanded binomial coefficients involving -1 or -2, e.g. \( \binom{-2}{1} \) are not sufficient for the M1.]

[f.t. is on \( A, B, C, D, E. \)]

[Apply this scheme to attempts to expand \( (9x^2 + 4)(1+2x)^{-1}(x - 2)^{-2} \), giving M1A2√A1 for a correct product of expansions and A1 for multiplying out and reaching the given answer correctly.]

[Allow attempts to multiply out \( (1 + 2x)(x - 2)^2 \) \( (1 - x + 5x^2) \), giving B1 for reduction to a product of two expressions correct up to their terms in \( x^2 \), M1 for attempting to multiply out as far as terms in \( x^2 \), A1 for a correct expansion, and A1 for obtaining \( 9x^2 + 4 \) correctly.]

[SR: \( B \) or \( C \) omitted from the form of partial fractions. In part (i) give the first B1, and M1 for the use of a relevant method to obtain \( A, B, \) or \( C \), but no further marks. In part (ii) only the M1 and A1√ for an unsimplified sum are available.]

[SR: \( E \) omitted from the form of partial fractions. In part (i) give the first B1, and M1 for the use of a relevant method to obtain \( A \) or \( D \), but no further marks. In part (ii) award M1A2√A1 as in the scheme.]

OR Differentiate and evaluate \( f(0) \) and \( f'(0) \)
Obtain \( f(0) = 1 \) and \( f'(0) = -1 \)
Differentiate and obtain \( f''(0) = 10 \)
Form the Maclaurin expansion and obtain the given answer correctly
7 (i) State or imply that \( \frac{dx}{dt} = k(100 - x) \) B1
Justify \( k = 0.02 \) B1

(ii) Separate variables and attempt to integrate \( \frac{1}{100 - x} \) M1
Obtain term – ln \((100 - x)\), or equivalent A1
Obtain term 0.02\(t\), or equivalent A1
Use \(x = 5\), \(t = 0\) to evaluate a constant, or as limits M1
Obtain correct answer in any form, e.g. \(-\ln(100 - x) = 0.02t - \ln95\) A1
Rearrange to give \(x\) in terms of \(t\) in any correct form, e.g. \(x = 100 - 95\exp(-0.02t)\) A1

[6]

[SR: \(-\ln(100 - x)\) for \(-\ln(100 - x)\). If no other error and \(x = 100 - 95\exp(0.02t)\) or equivalent obtained, give M1A0A1M1A0A1√]

(iii) State that \(x\) tends to 100 as \(t\) becomes very large B1

8 (i) State derivative \(1 - \frac{2}{x} - \frac{2}{x^2}\), or equivalent B1
Equate 2-term derivative to zero and attempt to solve for \(x\) M1
Obtain coordinates of stationary point \((2, \ln 2 + 1)\), or equivalent A1+A1
Determine by any method that it is a minimum point, with no incorrect work seen A1

[5]

(ii) State or imply the equation \(\alpha = \frac{2}{3 - \ln \alpha}\) B1
Rearrange this as \(3 = \ln \alpha + \frac{2}{\alpha}\) (or vice versa) B1

[2]

(iii) Use the iterative formula correctly at least once M1
Obtain final answer 0.56 A1
Show sufficient iterations to justify its accuracy to 2 d.p., or show there is a sign change in the interval \((0.555, 0.565)\) A1

[3]

9 (i) State or imply a correct normal vector to either plane, e.g. \(\hat{i} + 2\hat{j} - 2\hat{k}\) or \(2\hat{i} - 3\hat{j} + 6\hat{k}\) B1
Carry out correct process for evaluating the scalar product of both the normal vectors M1
Using the correct process for the moduli, divide the scalar product of the two normals by the product of their moduli and evaluate the inverse cosine of the result M1
Obtain answer \(40.4°\) (or \(40.3°\)) or \(0.705\) (or \(0.704\)) radians A1
[Allow the obtuse answer \(139.6°\) or \(2.44\) radians] [4]
(ii) EITHER Carry out a complete strategy for finding a point on $l$ M1
Obtain such a point e.g. $(0, 3, 2)$ A1

EITHER Set up two equations for a direction vector $a\mathbf{i} + bj + ck$ of $l$, e.g. $a + 2b - 2c = 0$ B1
and $2a - 3b + 6c = 0$
Solve for one ratio, e.g. $a:b$ M1
Obtain $a:b:c = 6:-10:-7$, or equivalent A1
State a correct answer, e.g. $r = 3j + 2k + \lambda (6i - 10j - 7k)$ A1√

OR Obtain a second point on $l$, e.g. $(6, -7, -5)$ A1
Subtract position vectors to obtain a direction vector for $l$ M1
Obtain $6i - 10j - 7k$, or equivalent A1
State a correct answer, e.g. $r = 3j + 2k + \lambda (6i - 10j - 7k)$ A1√

OR Attempt to find the vector product of the two normal vectors M1
Obtain two correct components A1
Obtain $6i - 10j - 7k$, or equivalent A1
State a correct answer, e.g. $r = 3j + 2k + \lambda (6i - 10j - 7k)$ A1√

OR Express one variable in terms of a second M1
Obtain a correct simplified expression, e.g. $x = (9 - 3y)/5$ A1
Express the same variable in terms of the third and form a three term equation M1
Incorporate a correct simplified expression, e.g. $x = (12 - 6z)/7$ in this equation A1
Form a vector equation for the line M1
State a correct answer, e.g. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -5/3 \\ -7/6 \end{pmatrix}$, or equivalent A1√

OR Express one variable in terms of a second M1
Obtain a correct simplified expression, e.g. $y = (9 - 5x)/3$ A1
Express the third variable in terms of the second M1
Obtain a correct simplified expression, e.g. $z = (12 - 7x)/6$ A1
Form a vector equation for the line M1
State a correct answer, e.g. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -5/3 \\ -7/6 \end{pmatrix}$, or equivalent A1√

10 (i) EITHER Make relevant use of the correct sin $2A$ formula M1
Make relevant use of the correct cos $2A$ formula M1
Derive the given result correctly A1

OR Make relevant use of the tan $2A$ formula M1
Make relevant use of $1 + \tan^2 A = \sec^2 A$ or $\cos^2 A + \sin^2 A = 1$ M1
Derive the given result correctly A1
(ii) State or imply indefinite integral is \( \ln \sin x \), or equivalent \( B1 \)
Substitute correct limits correctly \( M1 \)
Obtain given exact answer correctly \( A1 \)

[3]

(iii) \( EITHER \) State indefinite integral of \( \cos 2x \) is of the form \( k \ln \sin 2x \) \( M1 \)
State correct integral \( \frac{1}{2} \ln \sin 2x \) \( A1 \)
Substitute limits correctly throughout \( M1 \)
Obtain answer \( \frac{1}{4} \ln 3 \), or equivalent \( A1 \)

\( OR \) State or obtain indefinite integral of \( \csc 2x \) is of the form \( k \ln \tan x \), or equivalent \( M1 \)
State correct integral \( \frac{1}{2} \ln \tan x \), or equivalent \( A1 \)
Substitute limits correctly \( M1 \)
Obtain answer \( \frac{1}{4} \ln 3 \), or equivalent \( A1 \)

[4]
# Mechanics 1

## 1

|   | (i) | Tension is 8000 N or 800g
Accept 7840 N (from 9.8) or 7850 (from 9.81) | B1 | 1 |
<table>
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</thead>
<tbody>
<tr>
<td></td>
<td>(ii)</td>
<td>For using $P = \frac{\Delta W}{\Delta t}$ or $P = Tv$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Delta W = 8000 \times 20$ or $v = \frac{20}{50}$</td>
<td>A1</td>
<td></td>
</tr>
</tbody>
</table>
|   |     | Power applied is 3200 W
Accept 3140 W (from 9.8 or 9.81) | A1 | 3 |

**SR** (for candidates who omit $g$)  
(Max 2 out of 3)

$P = 800 \times 20 \div 50$  
B1  
Power applied is 320 W  
B1

## 2

<table>
<thead>
<tr>
<th></th>
<th>(i)</th>
<th>For resolving in the direction $PQ$</th>
<th>M1</th>
<th></th>
</tr>
</thead>
</table>
|   | (a) | Component is $2 \times 10\cos30^\circ - 6\cos60^\circ$  
or $14.3$ N or $10\sqrt{3} - 3$ N | A1 | 2 |
|   | (b) | Component is $\pm 6\cos30^\circ - 6\cos60^\circ$ or $\pm 5.20$ N  
or $\pm 3\sqrt{3}$ N | B1 | 1 |

**SR** (for candidates who resolve parallel to and perpendicular to the force of magnitude 6 N)  
(Max 2 out of 3)

For resolving in both directions  
M1  
For $X = 6 - 10\cos30^\circ$ or $-2.66$ N and  
$Y = 10 + 10\sin30^\circ$ or $15$ N  
A1

**SR** (for candidates who give a combined answer for (a) and (b))  
(Max 2 out of 3)

For resolving in both directions  
M1  
For $(6\cos30^\circ)i + (2 \times 10\cos30^\circ - 6\cos60^\circ)j$ or any vector equivalent  
A1

<table>
<thead>
<tr>
<th></th>
<th>(ii)</th>
<th>For using Magnitude $= \sqrt{ans(i)^2 + ans(ii)^2}$</th>
<th>M1</th>
<th></th>
</tr>
</thead>
</table>
|   |     | Magnitude is 15.2 N
ft only following sin/cos mix and for answer 5.66 N | A1 | 2 |

## 3

<table>
<thead>
<tr>
<th></th>
<th>(i)</th>
<th>Region under $v = 2t$ from $t = 0$ to $t = T$ indicated</th>
<th>B1</th>
<th>1</th>
</tr>
</thead>
</table>
|   | (ii) | For attempting to set up and solve an equation using  
area $\Delta = 16$  
or for using $s = \frac{1}{2} 2t^2$ | M1 |   |
|   |     | For $16 = \frac{1}{2} 2T^2$ | A1 |   |
|   |     | $T = 4$ | A1 | 3 |

**SR** (for candidates who find the height of the $\Delta$ but do not score M1)  
(Max 1 out of 3)

For $h/T = 2$ or $h = 2T$ or $v = 8$  
B1
<table>
<thead>
<tr>
<th>Question</th>
<th>Part</th>
<th>Solution</th>
</tr>
</thead>
</table>
| 4 | (i) | For differentiating $x$

$$\dot{x} = t + \frac{1}{10}t^2$$

Speed is $20\,\text{ms}^{-1}$

| | | A1 3 |
| 4 | (ii) | For attempting to solve $\ddot{x}(t) = 2\ddot{x}(0)$

$$\left(1 + \frac{1}{5}t = 2\right)$$

$t = 5$

| | | A1 3 |
| 5 | (i) | For resolving forces on any two of $A$, or $B$, or $A$ and $B$ combined ($T_1 = W_A + T_2, T_2 = W_B, T_1 = W_A + W_B$)

Tension in $S_1$ is 4 N or Tension in $S_2$ is 2 N
Accept 0.4g or 3.92 (from 9.8 or 9.81) for $T_1$
Tension in $S_2$ is 2 N or Tension in $S_1$ is 4 N
Accept 0.2g or 1.96 (from 9.8 or 9.81) for $T_2$

SR (for candidates who omit $g$) (Max 1 out of 3)

$T_1 = 0.4$ and $T_2 = 0.2$

| | | B1 |
| | | A1 3 |
| 5 | (ii) | For applying Newton’s second law to $A$, or to $B$, or to $A$ and $B$ combined

For any one of the equations $T + 2 - 0.4 = 0.2a$,
$2 - T - 0.2 = 0.2a$, $4 - 0.4 - 0.2 = 0.4a$

For a second of the above equations

For solving the simultaneous equations for $a$ and $T$

Acceleration is $8.5\,\text{ms}^{-2}$, tension is $0.1$ N
Accept 8.3 from 9.8 or 8.31 from 9.81

SR (for candidates who obtain only the ‘combined’ equation) (Max 3 out of 5)

For applying Newton’s second law to $A$ and $B$ combined

For $4 - 0.4 - 0.2 = 0.4a$

Acceleration is $8.5\,\text{ms}^{-2}$

A1 5
<p>| | | | | |</p>
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</thead>
<tbody>
<tr>
<td>6</td>
<td>(i)</td>
<td>For using $F = \mu R \text{ and } R = mg$ \hspace{1cm} (\text{Where } F = 0.025 \times 0.15 \times 10)</td>
<td>M1</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Frictional force is 0.0375 N or 3/80 N \hspace{1cm} Accept 0.0368 from 9.8 or 9.81</td>
<td>A1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>(ii)</td>
<td>For using $F = ma \text{ (-0.0375 = 0.15a) or } d = \mu g$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Deceleration is 0.25 ms$^{-2}$ \hspace{1cm} (or } a = - 0.25)</td>
<td>A.G.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(iii)</td>
<td>For using $s = ut + \frac{1}{2} at^2$ \hspace{1cm} (s = 5.5 \times 4 + \frac{1}{2} (-0.25)16)</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Distance AB is 20m</td>
<td>A1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>(iv)</td>
<td>For using $v^2 = u^2 + 2as$ \hspace{1cm} (v^2 = 3.5^2 - 2 \times 0.25 \times 20)</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Speed is 1.5 ms$^{-1}$ \hspace{1cm} (ft $\sqrt{(24.5 - (iii))/(2)}$)</td>
<td>A1 ft</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>(v)</td>
<td>Return dist. = $\frac{3.5^2}{2 \times 0.25}$ \hspace{1cm} or distance beyond A = $\frac{(iv)^2}{2 \times 0.25}$</td>
<td>M1</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Total distance is 44.5 m \hspace{1cm} (ft 24.5 + (iii) or 2((iv)$^2$ + (iii))</td>
<td>A1 ft</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>(i)</td>
<td>PE gain = $mg(2.5 \sin 60^\circ)$</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>For using KE = $\frac{1}{2} mv^2$</td>
<td>M1</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>For using the principle of conservation of energy \hspace{1cm} ($\frac{1}{2} m8^2 - \frac{1}{2} mv^2 = mg(2.5 \sin 60^\circ)$)</td>
<td>M1</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Alternative for the above 3 marks: \hspace{1cm} For using Newton’s Second Law or stating $a = -g \sin 60^\circ$ \hspace{1cm} $a = -8.66$ (may be implied) \hspace{1cm} For using $v^2 = u^2 + 2as$ \hspace{1cm} ($v^2 = 64 - 2 \times 8.66 \times 2.5$)</td>
<td>M1*</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Speed is 4.55 ms$^{-1}$ \hspace{1cm} Accept 4.64 from 9.8 or 9.81</td>
<td>A1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>(ii)</td>
<td>For using $\frac{1}{2} mu^2$ \hspace{1cm} $mg h_{\max}$ \hspace{1cm} ($\frac{1}{2} 8^2 &gt; 10 h_{\max}$)</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>For obtaining 3.2m</td>
<td>A.G.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(iii)</td>
<td>Energy is conserved or absence of friction or curve BC is smooth (or equivalent) and B and C are at the same height or the PE is the same at A and B (or equivalent)</td>
<td>B1</td>
<td>1</td>
</tr>
</tbody>
</table>
(iv) WD against friction is $1.4 \times 5.2$

B1

For WD = KE loss (or equivalent) used

M1

\[
1.4 \times 5.2 = \frac{1}{2} 0.4(8^2 - v^2) \quad \text{or}
\]

\[
1.4 \times 5.2 = \frac{1}{2} 0.4((i)^2 - v^2) + 0.4 \times 10(2.5 \sin 60^\circ)
\]

\[12.8 \text{ or } 4.14 + 8.66\]

A1

Alternative for the above 3 marks:

For using Newton’s Second Law

\[0.4g(2.5 \sin 60^\circ \div 5.2) - 1.4 = 0.4a \quad (a = 0.6636)\]

M1*

For using \(v^2 = u^2 + 2as\) with \(u \neq 0\)

\[v^2 = 4.55^2 + 2 \times 0.6636 \times 5.2\]

M1dep*

Speed is 5.25 ms\(^{-1}\)

A1 4
June 2003

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/05, 8719/05

MATHEMATICS AND HIGHER MATHEMATICS
Paper 5 (Mechanics 2)
Mechanics 2

1 The distance from the centre to the rod is $\sqrt{25^2 - 24^2}$
   B1

For taking moments about the centre of the ring or about the mid-point of the rod, or C.O.M. of frame
   (correct number of terms required in equation) M1

$$(1.5 + 0.6)\bar{x} = 0.6 \times 7 \text{ or } (1.5 + 0.6)(7 - \bar{x}) = 1.5 \times 7$$

$$1.5\bar{x} = 0.6(7 - \bar{x})$$  A1

Distance is 2cm
   A1

SR Allow M1 for $48.7 = (50\pi + 48)\bar{x}$

2 (i) $OQ = 4 \tan 20^\circ (=1.456)$  B1

   $OG = 1.5$  B1

   $G$ not between $O$ and $Q$ (all calculations correct)  B1

   3

(ii) Hemisphere does not fall on to its plane face  *B1 ft

   Because the moment about $P$ is clockwise or the centre of mass is to right of $PQ$
   (dep)* B1 ft

3 (i) Rope is at $30^\circ$ to wall, or beam is at $0^\circ$ to the horizontal or a correct trig. ratio used  B1

   For taking moments about $A$ or
   For taking moments about $P$ and resolving horizontally M1

   $$2.5T = 45g \times 3\cos 30^\circ \text{ or } 5H = 45g \times 3\cos 30^\circ \text{ and } H = T\sin 30^\circ$$

   Tension is 468 N  A1

   4

(ii) Horizontal component is 234 N  (ft $\frac{1}{2}T$)  B1 ft

   For resolving forces vertically ($V = 45g - T\cos 30^\circ$)  M1

   Magnitude of vertical component is 45 N  A1 ft

   SR angle incorrect (i) B0, M1, A1 ft A0, (ii) B1 ft ($T$ and angle), M1, A0  3
4 (i) For using Newton's second law with \( a = \mathbf{v} \frac{dv}{dx} \)
\[
- \frac{1}{3\mathbf{v}} = 0.2v \frac{dv}{dx}
\]
\[
3\mathbf{v}^2 \frac{dv}{dx} = -5 \text{ from correct working}
\]
\[M1\]
\[A1\]
\[A1\]

(ii) For separating the variables and attempting to integrate
\[
\mathbf{v}^3 = (A) - 5x
\]
For using \( x = 0 \) and \( \mathbf{v} = 4 \) to find \( A \), and then substituting \( x = 7.4 \) (or equivalent using limits)
\[M1\]
\[\mathbf{v} = 3\]
\[A1\]

5 (i) For resolving forces vertically (3 term equation)
\[
T \cos 60^\circ + 0.5 \times 10 = 8
\]
Tension is 6 N
\[M1\]
\[A1\]

(ii) Radius of circle is \( 9 \sin 60^\circ \) (7.7942)
\[B1\]
For using Newton's second law horizontally with \( a = \frac{v^2}{r} \)
\[
6 \sin 60^\circ = 0.5 \frac{v^2}{(9 \sin 60^\circ)}
\]
\[A1 \text{ ft}\]
Alternative for the above 2 marks:
For using Newton's second law perpendicular to the string with \( a = \frac{v^2}{r} \)
\[
(8 - 0.5 \times 10) \sin 60^\circ = 0.5 \frac{v^2}{(9 \sin 60^\circ)} \cos 60^\circ
\]
\[A1 \text{ ft}\]
Speed is 9 ms\(^{-1}\)
\[A1\]

\[NB\] Use of \( mr\omega^2 \), the M1 is withheld until \( v = r\omega \) is used
\[SR\] Lift perpendicular to the string:
(i) \( 8 \sin 60^\circ = 0.5g + T \cos 60^\circ \rightarrow T = 3.86 \): M1, A1, A1 (-1 MR) (2 out of 3 max);
(ii) \( 3.86 \sin 60^\circ + 8 \cos 60^\circ = \frac{0.5v^2}{9 \sin 60^\circ} \): B1, M1, A1, A1 (-1 MR) (3 out of 4 max)
\[\Rightarrow 10.7\]
6  (i) For using \( y = y_0 t - \frac{1}{2} gt^2 \) with \( y = 0 \) and \( t = 10 \) or
\[
\dot{y} = \dot{y}_0 - gt \text{ with } \dot{y} = 0 \text{ and } t = 5 \quad \text{M1}
\]
\[
0 = 60\sin\alpha \times 10 - \frac{1}{2} \times 10 \times 10^2 \text{ or } 0 = 60\sin\alpha \times 10 \times 5
\]
\[
\alpha = 56.4^\circ \quad \text{A1}
\]

(ii) For substituting \( t = 5 \) into \( y = y_0 t - \frac{1}{2} gt^2 \) or \( \dot{y} = 0 \) into
\[
\dot{y}^2 = \dot{y}_0^2 - 2gy \text{ or } \dot{y} = 0 \text{ and } t = 5 \text{ into } y = \frac{\dot{y}_0 + \dot{y}}{2} t \quad \text{M1}
\]

Greatest height is 125m

A1

(iii) \( \dot{y} = 60\sin\alpha - gT \)
\[
\dot{x} = 60\cos\alpha \quad \text{B1}
\]

For attempting to solve \( \dot{x} = \dot{y} \), or a complete method

M1

for an equation in \( T \) using \( \dot{x} = \dot{y} \)
\[
T = 1.68 \quad \text{A1}
\]

NB. Use of \( \dot{y}_0 = 60 \) in (i) and (ii) is M0
7 (i) For using $T = \frac{Lx}{L} \left( \frac{130 \times 3}{10} \text{ or } \frac{130 \times 1.5}{5} \right)$  

Tension is 39 N  

A1  

2  

(ii) For resolving forces vertically ($mg = 2 \times 39 \times \frac{5}{13}$)  

Mass is 3kg  

A1  

2  

(iii) Extension = 20 - 10 (or 10 - 5)  

For using $EPE = \frac{Lx^2}{2L}$  

$L$ must be 10 or 5; must be attempt at extension, e.g. $x = 20$ or $x = 8 - 2.5$ is M0)  

$EPE = \frac{130 \times 10^2}{2 \times 10} \text{ or } EPE = 2 \times \frac{130 \times 5^2}{2 \times 5}$  

(Allow M1 only for $x = 2$ or 3)  

M1  

EPE is 650 J (ft attempted extension in lowest position)  

A1  

3  

(iv) Change in GPE = 3 x 10 x 8  

For using the principle of conservation of energy with KE, GPE and EPE all represented  

$650 = \frac{1}{2}mv^2 + 3 \times 10 \times 8 + \frac{130 \times 2^2}{2 \times 10}$  

A1  

4  

Speed is 16 ms$^{-1}$  

A1
June 2003

GCE A AND AS LEVEL
AICE

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/06, 0390/06

MATHEMATICS
Paper 6 (Probability and Statistics 1)
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<tbody>
<tr>
<td><strong>1</strong></td>
<td>(i)</td>
<td>False zero</td>
<td>B1 1 Or any sensible answer</td>
</tr>
<tr>
<td>(ii)</td>
<td>(a) Stem</td>
<td>Leaf</td>
<td>B1 B1 B1 For correct stem, i.e. not 30, 40, 50 etc. For correct leaf, must be sorted</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>Key 3│4 rep 34, or stem width = 10</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>B1 3 For key, NB 30│4 rep 34 gets B1 here</td>
</tr>
<tr>
<td>(b)</td>
<td>79</td>
<td>B1 ft 1 For correct answer, only ft from a sorted stem and leaf diagram</td>
<td></td>
</tr>
<tr>
<td><strong>2</strong></td>
<td>(i)</td>
<td>P(N, N) = 3/10 x 7/9 Mult. By 2 = 7/15 AG</td>
<td>M1 A1 2 For multiplying 2 relevant possibilities For obtaining given answer legitimately</td>
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<tr>
<td></td>
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<td>OR Total ways 10C2 (= 45) Total 1 of each 7C1 x 3C1 (= 21) Prob = 21/45 = 7/15 AG</td>
<td>M1 A1 2 For both totals For obtaining correct answer</td>
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<td></td>
<td>(ii)</td>
<td>P(N, N) – 3/10 x 2/9 (= 1/15)</td>
<td>M1 M1 3 For 2 correct numbers multiplied together, can be implied For 2 correct numbers multiplied together or subtracting from 1 All correct. Table correct and no working gets 3/3</td>
</tr>
<tr>
<td></td>
<td>P(N, N) = 7/10 x 6/9 (= 7/15)</td>
<td>x 0 1 2</td>
<td></td>
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<tr>
<td></td>
<td>P(X=x) 7/15 15/15 15/15</td>
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<td></td>
<td>(iii)</td>
<td>E(X) = 1 x 7/15 + 2 x 1/15 = 3/5</td>
<td>B1 ft 1 For correct answer or equivalent. Only ft if ∑ p = 1</td>
</tr>
<tr>
<td><strong>3</strong></td>
<td>(i)</td>
<td>P(X &gt; 120)</td>
<td>M1 M1 A1 3 For standardising with or without the √, 17.2^2, but no cc. For finding the correct area, 1 – their Φ(z), NOT Φ(1 – their z(0.4651)) For correct answer</td>
</tr>
<tr>
<td></td>
<td>= 1 - Φ(120 – 112)</td>
<td>120 – 112 17.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 1 - Φ (0.4651)</td>
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<td>= 1 - 0.6790 = 0.321</td>
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<tr>
<td>ii) ( z = -0.842 )</td>
<td>( -0.842 = \frac{103 - 115}{\sigma} )</td>
<td>( \sigma = 14.3 )</td>
<td>B1 M1 A1 3 For solving an equation involving their ( z ) or ( z = 0.7881 ) or ( 0.5793 ) only, 103, 115 and ( \sigma ) or ( \sqrt{\sigma} ) or ( \sigma^2 ), i.e. must have used tables For correct answer</td>
</tr>
<tr>
<td>4 (i) ( (0.7)^{24} \times (0.3)^{5} \times 30C_{24} )</td>
<td>( = 0.0829 )</td>
<td>OR normal approx. ( P(24) = \Phi \left( \frac{24.5 - 21}{\sqrt{6.3}} \right) ) - ( \Phi \left( \frac{23.5 - 21}{\sqrt{6.3}} \right) )</td>
<td>M1 A1 2 For relevant binomial calculation For correct answer</td>
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<tr>
<td>ii) ( \mu = 30 \times 0.7 = 21, ) ( \sigma^2 = 30 \times 0.7 \times 0.3 = 6.3 )</td>
<td>( P(&lt; 20) = \Phi \left( \frac{19.5 - 21}{\sqrt{6.3}} \right) ) = ( \Phi (-0.5976) )</td>
<td>( = 1 - 0.7251 = 0.275 )</td>
<td>B1 M1 A1 5 For 21 and 6.3 seen For standardising process, must have ( \sqrt{\cdot} ), can be + or – For continuity correction 19.5 or 20.5 For using 1 - some area found from tables For correct answer</td>
</tr>
<tr>
<td>5 (i) ( 6C_3 \times 4C_2 = 120 )</td>
<td></td>
<td></td>
<td>M1 2 For multiplying 2 combinations together, not adding, no perms, ( 10C_3 \times 10C_2 ) or ( 5C_3 \times 5C_2 ) would get M1 For answer 120</td>
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<tr>
<td>ii) ( 6C_4 \times 4C_1 = (= 60) )</td>
<td></td>
<td></td>
<td>M1 3 For reasonable attempt on option 4M 1W, or 5M, 0W, can have + here and perms For other option attempt For correct answer</td>
</tr>
<tr>
<td>(i) ( 6C_5 \times 4C_0 = (= 6) )</td>
<td></td>
<td></td>
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<tr>
<td>Answer = 186</td>
<td></td>
<td></td>
<td>A1 3 For correct answer</td>
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<tr>
<td>(i) Man and woman both on ( 5C_2 \times 3C_1 = (= 30) )</td>
<td></td>
<td></td>
<td>M1 3 For finding number of ways of the man and woman being on together, need not be evaluated but must be multiplied For subtracting a relevant number from their (i) For correct answer</td>
</tr>
</tbody>
</table>
### Mark Scheme

#### Syllabus Paper

#### A AND AS LEVEL – JUNE 2003

<table>
<thead>
<tr>
<th>Question</th>
<th>Statement</th>
<th>Mark</th>
<th>Description</th>
</tr>
</thead>
</table>

6. (i) \( P(G) = \frac{\text{number of g'parents}}{\text{total people}} \)

- \( \frac{6}{16} = \frac{3}{8} \)

M1

A1

For appreciating total g’parents/total people, can be implied

(ii) \( P(H1, G) + P(H2, G) + P(H3, G) \)

- \( \frac{1}{3} \times \frac{2}{7} + \frac{1}{3} \times \frac{3}{7} + \frac{1}{3} \times \frac{1}{2} = \frac{17}{42} \)

(= 0.405)

M1

B1

For any correct 2-factor product, need not be evaluated

For addition of 3 relevant 2-factor products

For correct answer or equivalent

(iii) \( P(H1 | G) + P(H2 | G) \)

- \( \frac{2}{17} + \frac{3}{17} = \frac{10}{17} \)

M1

A1

For summing exactly 2 probability options

For dividing by answer to (ii), only if not multiplied as well, and \( p \) must be < 1

For one correct probability

For correct answer or equivalent

OR \( P(H3 | G) = \frac{7}{17} \)

Answer = \( 1 - \frac{7}{17} \)

= \( \frac{10}{17} \)

M1

M1

A1

For finding prob. options no parents

For subt. from 1

For correct answer

7. (i) Mean = \( \frac{\sum fx}{\sum f} \)

- \( 18.4 \)

M1

A1

For using their mid-intervals (not end points or class widths)

For using \( \frac{\sum f x^2}{\sum f} \) any \( x \)

For correct answer, cwo, 18.4 no wkg

3/3
\[
\text{sd} = \sqrt{\left(2.5^2 \times 11 + 7.5^2 \times 20 + 15^2 \times 32 + 25^2 \times 18 + 35^2 \times 10 + 55^2 \times 6 \right) / 97 - \text{mean}^2} = 13.3
\]

- **M1** For using \( \frac{\sum fx^2}{\sum f} - (\text{their mean})^2 \) or equivalent, no \( \sqrt{\text{needed, not}} \) \( (\sum fx)^2 / \sum f \)
- **A1** For correct answer

(ii) Freq. densities: 2.2, 4.0, 3.2, 1.8, 1.0, 0.2

- **M1** For attempting a frequency density of some sort (or scaled frequency), can be upside down but not multiplied
- **A1** For correct heights on the graph
- **B1** For correct bars on uniform horiz. scale, i.e. from 0 to 5 etc.
- **B1** For freq. density or scaled freq. labelled on vertical axis, time or mins on horiz., ‘class width’ is not enough
<table>
<thead>
<tr>
<th>MARK SCHEME</th>
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<tbody>
<tr>
<td>MAXIMUM MARK: 50</td>
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<tr>
<td>SYLLABUS/COMPONENT: 9709/07, 8719/07</td>
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<tr>
<td>MATHEMATICS AND HIGHER MATHEMATICS</td>
</tr>
<tr>
<td>Paper 7 (Probability and Statistics 2)</td>
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<tr>
<td><strong>1 (i)</strong></td>
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<tr>
<td><strong>1 (ii)</strong></td>
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<tr>
<td><strong>2</strong></td>
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<tr>
<td><strong>3 (i)</strong></td>
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<td><strong>3 (ii)</strong></td>
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<td></td>
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<tr>
<td><strong>4 (i)</strong></td>
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<table>
<thead>
<tr>
<th>Question</th>
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<th>Notes</th>
</tr>
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<tbody>
<tr>
<td>(ii)</td>
<td>$E(X) = \int_0^x (x - \frac{1}{2}x^2),dx = \left[\frac{x^2}{2} - \frac{x^3}{6}\right]_0^x$</td>
<td>M1</td>
<td>For evaluating their $\int xf(x),dx$</td>
</tr>
<tr>
<td></td>
<td>$= \frac{2}{3}$</td>
<td>A1</td>
<td>2</td>
</tr>
<tr>
<td>(iii)</td>
<td>$m - \frac{m^2}{4} = 0.5$</td>
<td>M1</td>
<td>For equating their $\int f(x),dx$ to 0.5</td>
</tr>
<tr>
<td></td>
<td>$m = 0.586 (2-\sqrt{2})$</td>
<td>A1</td>
<td>3</td>
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</tbody>
</table>

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<tr>
<td>5 (i)</td>
<td>$P(X &lt; 1.7) = \Phi\left(\frac{1.7 - 2.1}{0.9/\sqrt{20}}\right)$</td>
<td>B1</td>
<td>For identifying prob Type I error</td>
</tr>
<tr>
<td></td>
<td>$= 1 - \Phi(1.9876)$</td>
<td>M1</td>
<td>For standardising</td>
</tr>
<tr>
<td></td>
<td>$= 0.0234$</td>
<td>A1</td>
<td>4</td>
</tr>
<tr>
<td>(ii)</td>
<td>$P($Type II error$) = P(X &gt; 1.7)$</td>
<td>B1</td>
<td>For identifying prob for Type II error</td>
</tr>
<tr>
<td></td>
<td>$= 1 - \Phi\left(\frac{1.7 - 1.5}{0.9/\sqrt{20}}\right)$</td>
<td>M1</td>
<td>For standardising using 1.5 and their 1.7</td>
</tr>
<tr>
<td></td>
<td>$= 1 - \Phi(0.9938) = 0.160$</td>
<td>A1</td>
<td>4</td>
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<tbody>
<tr>
<td>6 (i)</td>
<td>$\lambda = 1.25$</td>
<td>M1</td>
<td>For attempting to find new $\lambda$ and using it</td>
</tr>
<tr>
<td></td>
<td>$P(X &lt; 4) = e^{-1.25 \left(1 + 1.25 + \frac{1.25^2}{2} + \frac{1.25^3}{6}\right)}$</td>
<td>M1</td>
<td>For summing $P((0, 1, 2, 3))$ or $P(0, 1, 2, 3, 4)$ using a Poisson expression</td>
</tr>
<tr>
<td></td>
<td>$= 0.962$</td>
<td>A1</td>
<td>3</td>
</tr>
<tr>
<td>(ii)</td>
<td>$X \sim N(182.5, 182.5)$</td>
<td>B1</td>
<td>For correct mean and variance</td>
</tr>
<tr>
<td></td>
<td>$P(&gt;200$ breakdowns$) = 1 - \Phi\left(\frac{200.5 - 182.5}{\sqrt{182.5}}\right)$</td>
<td>M1</td>
<td>For standardising process with or without continuity correction</td>
</tr>
<tr>
<td></td>
<td>$= 1 - \Phi(1.332)$</td>
<td>A1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>$= 0.0915 (0.0914)$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td>$\lambda = 5$ for phone calls</td>
<td>B1</td>
<td></td>
</tr>
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<td></td>
<td>$\lambda = 6.25$ for total</td>
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<tr>
<td></td>
<td>$P(X = 4) = e^{-6.25 \left(\frac{6.25^4}{4!}\right)}$</td>
<td>M1</td>
<td>For summing their two $\lambda$ s and using a Poisson expression OR alt. method using sep. distributions 5 terms req.</td>
</tr>
<tr>
<td></td>
<td>$= 0.123$</td>
<td>A1</td>
<td>3</td>
</tr>
</tbody>
</table>
| 7 (i) | \[ \begin{align*} & \text{20 of } A \sim A^* \\
& \sim \text{N}(401, 20 \times 0.15^2) \\
& \sim \text{N}(401, 0.45) \\
& \text{20 of } B \sim B^* \sim \text{N}(401, 1.458) \\
& A^* - B^* \sim \text{N}(0, 1.908) \end{align*} \] |
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<tbody>
<tr>
<td></td>
<td>[ P(A^* - B^* &gt; 2) = 1 - \Phi \left( \frac{2 - 0}{\sqrt{1.908}} \right) ]</td>
</tr>
<tr>
<td></td>
<td>[ = 1 - \Phi (1.4479) ]</td>
</tr>
<tr>
<td></td>
<td>[ = 0.0738 ]</td>
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</tbody>
</table>
|       | **B1** For correct mean for either \[ A^* - B^* \sim \text{N}(0, 1.908) \] \[ P(A^* - B^* > 2) \]
|       | **B1** For variance \( 20 \times 0.15^2 \) or \( 20 \times 0.27^2 \) \[ \text{N}(401, 0.45) \]
|       | **M1** For adding their two variances \[ \text{N}(401, 1.458) \]
|       | **M1** For consideration of their \[ A^* - B^* > 2 \]
|       | **M1** For standardising and finding correct area \[ \Phi (1.4479) \]
|       | **A1** 6 For correct answer \[ 0.0738 \]

**OR**

| 7 (i) | \[ \begin{align*} & \overline{A} \sim \text{N}(20.05, 0.15^2/20), \\
& \overline{B} \sim \text{N}(20.05, 0.27^2/20) \\
& \overline{A} - \overline{B} \sim \text{N}(0, 0.00477) \end{align*} \] |
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<tbody>
<tr>
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<td>[ P(\overline{A} - \overline{B} &gt; 0.1) = 1 - \Phi \left( \frac{0.1 - 0}{\sqrt{0.00477}} \right) ]</td>
</tr>
<tr>
<td></td>
<td>[ = 1 - \Phi (0.1) ]</td>
</tr>
<tr>
<td></td>
<td>[ = 0.0738 ]</td>
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</tbody>
</table>
|       | **B1** For correct mean for either \[ \overline{A} \sim \text{N}(20.05, 0.15^2/20) \]
|       | **B1** For variance \( 0.15^2/20 \) or \( 0.27^2/20 \) \[ \text{N}(20.05, 0.27^2/20) \]
|       | **M1** For adding their variances \[ \text{N}(0, 0.00477) \]
|       | **M1** For consideration of their \[ \overline{A} - \overline{B} > 0.1 \]
|       | **M1** For standardising and finding correct area \[ \Phi (0.1) \]
|       | **A1** 6 For correct answer \[ 0.0738 \]

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<thead>
<tr>
<th>(ii)</th>
<th>[ 1.96 = \frac{20.07 - 20.05}{(0.15/\sqrt{n})} ]</th>
</tr>
</thead>
</table>
|      | **M1** For an equation of correct form on RHS involving \( \sqrt{n} \) \[ 1.96 \]
|      | **B1** For 1.96 used \[ 20.07 - 20.05 \]
|      | **M1** For solving an equation of correct form (any z) \[ (0.15/\sqrt{n}) \]
|      | **A1** 4 For correct answer \[ n = 216 \]